

# Precalculus Yearlong Mathematics Map

**Resources:** Approved from Board of Education

**Assessments:** District Benchmark Assessments

**Common Core State Standards – Standards for Mathematical Practice:**

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| <ul style="list-style-type: none"> <li>1. Make sense of problems and persevere in solving them.</li> <li>3. Construct viable arguments and critique the reasoning of others.</li> <li>5. Use appropriate tools strategically.</li> <li>7. Look for and make use of structure.</li> </ul> | <ul style="list-style-type: none"> <li>2. Reason abstractly and quantitatively.</li> <li>4. Model with mathematics.</li> <li>6. Attend to precision.</li> <li>8. Look for and express regularity in repeated reasoning.</li> </ul> |
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Conceptual Category	Domain	Cluster	Common Core Standard	Content	Skills	Academic Vocabulary
N	CN	Perform arithmetic operations with complex numbers.	N-CN.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.	Complex numbers - Conjugate	N-CN.3 Find the conjugate of a complex number	moduli, rationalizing the denominator
N	CN	Perform arithmetic operations with complex numbers.	N-CN.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.	Complex numbers - Conjugate	N-CN.3 Find moduli and quotients of complex numbers using conjugates	moduli
N	CN	Represent complex numbers and their operations on the complex plane.	N-CN.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.	Complex numbers - Complex plane	N-CN.4 Represent complex numbers on the complex plane in rectangular and polar form	polar coordinates, rectangular coordinates, lemniscate, limacon, rose, cardioid
N	CN	Represent complex numbers and their operations on the complex plane.	N-CN.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.	Complex numbers - Complex plane	N-CN.4 Explain why the rectangular and polar forms of a given complex number represent the same number	equivalent forms, unit conversions
N	CN	Represent complex numbers and their operations on the complex plane.	N-CN.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}i)^3 = 8$ because $(-1 + \sqrt{3}i)$ has modulus 2 and argument $120^\circ$ .	Complex numbers - Complex plane	N-CN.5 Utilize properties to add, subtract, multiply, and conjugate complex numbers	Operations with variables

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N	CN	Represent complex numbers and their operations on the complex plane.	N-CN.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}i)^3 = 8$ because $(-1 + \sqrt{3}i)$ has modulus 2 and argument $120^\circ$ .	Complex numbers - Complex plane	N-CN.5 Represent complex computations geometrically on a complex plane (Calculus)	
N	CN	Represent complex numbers and their operations on the complex plane.	N-CN.6 (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.	Complex numbers - Complex plane	N-CN.6 Calculate the distance between numbers in the complex plane as the modulus of the difference (Calculus)	
N	CN	Represent complex numbers and their operations on the complex plane.	N-CN.6 (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.	Complex numbers - Complex plane	N-CN.6 Calculate the midpoint of a segment as the average of the numbers at its endpoints (Calculus)	
N	VM	Represent and model with vector quantities.	N-VM.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $v$ , $ v $ , $\ v\ $ , $\mathbf{v}$ ).	Vectors	N-VM.1 Represent vector quantities by directed line segments with appropriate symbols for the vectors and their magnitudes	Distance formula, translations, magnitude, vector, component
N	VM	Represent and model with vector quantities.	N-VM.2 (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.	Vectors	N-VM.2 Find the components of a vector	vector, component
N	VM	Represent and model with vector quantities.	N-VM.3 (+) Solve problems involving velocity and other quantities that can be represented by vectors.	Vectors	N-VM.3 Solve problems involving velocity and other quantities that can be represented by vectors	vector
N	VM	Perform operations on vectors.	N-VM.4 (+) Add and subtract vectors.	Vectors	N-VM.4	
N	VM	Perform operations on vectors.	N-VM.4a Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.	Vectors - Operations	N-VM.4a Add vectors using their components	vector, component

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N	VM	Perform operations on vectors.	N-VM.4a Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.	Vectors - Operations	N-VM 4a Add vectors using the parallelogram rule	vector, component
N	VM	Perform operations on vectors.	N-VM.4b Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.	Vectors - Operations	N-VM.4b Determine the magnitude and direction of the sum of two vectors	trigonometric properties of angles, magnitude, vector, component
N	VM	Perform operations on vectors.	N-VM.4c Understand vector subtraction $v - w$ as $v + (-w)$ , where $-w$ is the additive inverse of $w$ , with the same magnitude as $w$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.	Vectors - Operations	N-VM.4c Subtract vectors using their components	vector, component
N	VM	Perform operations on vectors.	N-VM.4c Understand vector subtraction $v - w$ as $v + (-w)$ , where $-w$ is the additive inverse of $w$ , with the same magnitude as $w$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.	Vectors - Operations	N-VM.4c Subtract vectors graphically	vector
N	VM	Perform operations on vectors.	N-VM.5 (+) Multiply a vector by a scalar.	Vectors - Operations	N-VM.5	
N	VM	Perform operations on vectors.	N-VM.5a Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$ .	Vectors - Operations	N-VM.5a Multiply a vector and a scalar using the vector components	scalar, vector
N	VM	Perform operations on vectors.	N-VM.5a Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$ .	Vectors - Operations	N-VM.5a Multiply a vector and a scalar graphically	scalar, vector

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N	VM	Perform operations on vectors.	N-VM.5b Compute the magnitude of a scalar multiple $cv$ using $  cv   =  c v$ . Compute the direction of $cv$ knowing that when $ c v \neq 0$ , the direction of $cv$ is either along $v$ (for $c > 0$ ) or against $v$ (for $c < 0$ ).	Vectors - Operations	N-VM.5b Compute the direction and magnitude of a scalar multiple	vector, scalar multiple, magnitude
N	VM	Perform operations on matrices and use matrices in applications.	N-VM.6 (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.	Matrices - Operations	N-VM.6 Represent and manipulate data using matrices	matrix
N	VM	Perform operations on matrices and use matrices in applications.	N-VM.7 (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.	Matrices - Operations	N-VM.7 Multiply matrices by scalars to produce new matrices	distribution, scalar, matrix
N	VM	Perform operations on matrices and use matrices in applications.	N-VM.8 (+) Add, subtract, and multiply matrices of appropriate dimensions.	Matrices - Operations	N-VM.8 Add, subtract, and multiply matrices of appropriate dimensions	dimensions, matrix
N	VM	Perform operations on matrices and use matrices in applications.	N-VM.9 (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.	Matrices - Operations	N-VM.9 Prove commutative, associative, and distributive properties of matrices	commutative, associative, distributive properties from algebra, matrices
N	VM	Perform operations on matrices and use matrices in applications.	N-VM.10 (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.	Matrices - Operations	N-VM.10 Explain the role of zero and identity matrices in matrix addition and multiplication	matrices, square matrices

Conceptual Category	Domain	Cluster	Common Core Standard	Content	Skills	Academic Vocabulary
N	VM	Perform operations on matrices and use matrices in applications.	N-VM.10 (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.	Matrices - Operations	N-VM.10 Find the determinant of a square matrix	determinant, matrix
N	VM	Perform operations on matrices and use matrices in applications.	N-VM.11 (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.	Matrices - Operations	N-VM.11 Multiply a vector by a matrix of suitable dimensions to produce another vector	vector, dimensions, matrix
N	VM	Perform operations on matrices and use matrices in applications.	N-VM.11 (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.	Matrices - Operations	N-VM.11 Transform vectors using matrices	transform, vector, matrix, distribution
N	VM	Perform operations on matrices and use matrices in applications.	N-VM.12 (+) Work with $2 \times 2$ matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area.	Matrices - Operations	N-VM.12 Transform $2 \times 2$ matrices (Calculus)	transform, matrix, square matrices
N	VM	Perform operations on matrices and use matrices in applications.	N-VM.12 (+) Work with $2 \times 2$ matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area.	Matrices - Operations	N-VM.12 Interpret the absolute value of the determinant in terms of area (Calculus)	determinant, matrix
A	REI	Solve systems of equations.	A-REI.8 (+) Represent a system of linear equations as a single matrix equation in a vector variable.	Matrices - Solving Systems of equations	A-REI.8 Represent a system of linear equations as a single matrix equation in a vector variable	matrix, vector
A	REI	Solve systems of equations.	A-REI.9 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension $3 \times 3$ or greater).	Matrices - Solving Systems of equations	A-REI.9 Solve systems of linear equations using the inverse of a matrix	matrices, invertible

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F	IF	Analyze functions using different representations.	F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★	Functions - Analysis	F-IF.7	
F	IF	Analyze functions using different representations.	F-IF.7d (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.	Functions - Analysis	F-IF.7d Identify zeros through graphing rational functions	x-intercepts, interval notation, oblique asymptote, average rate of change
F	IF	Analyze functions using different representations.	F-IF.7d (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.	Functions - Analysis	F-IF.7d Identify asymptotes through graphing rational functions	dividing by zero, interval notation, oblique asymptote, average rate of change
F	IF	Analyze functions using different representations.	F-IF.7d (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.	Functions - Analysis	F-IF.7d Recognize end behavior through graphing rational functions	even and odd degree, interval notation, oblique asymptote, average rate of change
F	BF	Build a function that models a relationship between two quantities.	F-BF.1c (+) Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.	Functions - Composition	F-BF.1c Compose functions	Substitution
F	BF	Build new functions from existing functions.	F-BF.4 Find inverse functions.	Building functions - Inverses	F-BF.4	
F	BF	Build new functions from existing functions.	F-BF.4b (+) Verify by composition that one function is the inverse of another.	Building functions - Inverses	F-BF.4b Verify functions are inverses through composition	one to one

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F	BF	Build new functions from existing functions.	F-BF.4c (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.	Building functions - Inverses	F-BF.4c Calculate the coordinates of an inverse function from a graph or table	one to one
F	BF	Build new functions from existing functions.	F-BF.4d (+) Produce an invertible function from a non-invertible function by restricting the domain.	Building functions - Inverses	F-BF.4d Produce an invertible function from a non-invertible function by restricting the domain	invertible, one to one
F	BF	Build new functions from existing functions.	F-BF.5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.	Building functions - Inverses	F-BF.5 Compare the characteristics of exponents and logarithms	
F	BF	Build new functions from existing functions.	F-BF.5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.	Building functions - Inverses	F-BF.5 Solve problems involving logarithms and exponents	logistic
F	TF	Extend the domain of trigonometric functions using the unit circle.	F-TF.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$ , $\pi/4$ and $\pi/6$ , and use the unit circle to express the values of sine, cosines, and tangent for $x$ , $\pi + x$ , and $2\pi - x$ in terms of their values for $x$ , where $x$ is any real number.	Trigonometric Functions - Special Triangles	F-TF.3 Geometrically determine the values of sine, cosine, and tangent for $\pi/3$ , $\pi/4$ and $\pi/6$ using special triangles	30-60-90, 45-45-90 triangles, cosecant, secant, cotangent
F	TF	Extend the domain of trigonometric functions using the unit circle.	F-TF.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$ , $\pi/4$ and $\pi/6$ , and use the unit circle to express the values of sine, cosines, and tangent for $x$ , $\pi + x$ , and $2\pi - x$ in terms of their values for $x$ , where $x$ is any real number.	Trigonometric Functions - Unit Circle	F-TF.3 Express the values of sine, cosine, and tangent for $x$ , $\pi + x$ , and $2\pi - x$ using the unit circle	cosecant, secant, cotangent, midline, phase shift
F	TF	Extend the domain of trigonometric functions using the unit circle.	F-TF.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	Trigonometric Functions - Unit Circle	F-TF.4 Explain even/odd symmetry and periodicity of trigonometric functions using the unit circle	fold along y axis, rotate 180 degrees, even symmetry, odd symmetry

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F	TF	Model periodic phenomena with trigonometric functions.	F-TF.6 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.	Trigonometric Functions - Inverse	F-TF.6 Construct the inverse of a trigonometric function through restricting the domain	one to one
F	TF	Model periodic phenomena with trigonometric functions.	F-TF.7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. ★	Trigonometric Functions - Modeling	F-TF.7 Model trigonometric equation word problems using inverses and technology	Solving inverse trigonometric equations, one-to-one, linear speed, angular speed
F	TF	Prove and apply trigonometric identities.	F-TF.9 (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.	Trigonometric Functions - Proving Identities	F-TF.9 Prove the addition and subtraction formulas for sine, cosine, and tangent	Substitution
F	TF	Prove and apply trigonometric identities.	F-TF.9 (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.	Trigonometric Functions - Solving equations	F-TF.9 Solve trigonometric problems using the addition and subtraction formulas for sine, cosine, and tangent	
G	SRT	Apply trigonometry to general triangles	G-SRT.9 (+) Derive the formula $a = \frac{1}{2} ab \sin C$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.	Trigonometric Functions - Similarity, right triangles, and trigonometry	G-SRT.9 Derive the formula $a = \frac{1}{2} ab \sin C$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.	law of sines, law of cosines, Heron's formula
G	SRT	Apply trigonometry to general triangles	G-SRT.10 (+) Prove the law of cosines and sines and use them to solve problems.	Trigonometric Functions - Similarity, right triangles, and trigonometry	G-SRT.10 Prove the law of cosines and sines and use them to solve problems.	law of sines, law of cosines
G	SRT	Apply trigonometry to general triangles	G-SRT.11 (+) Understand and apply the law of sines and law of cosines to find unknown measurements in right and non-right triangles. (e.g. Surveying problems, resultant forces)	Trigonometric Functions - Similarity, right triangles, and trigonometry	G-SRT.11 Understand and apply the law of sines and law of cosines to find unknown measurements in right and non-right triangles. (e.g. Surveying problems, resultant forces)	law of sines, law of cosines



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G	GPE	Translate between the geometric description and the equation for a conic section	G-GPE.3 (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.	Conic Sections - Forming Equations	G-GPE.3 Derive the equations of ellipses given the foci, using the fact that the sum or difference of distances from the foci is constant	circles, Pythagorean theorem
G	GPE	Translate between the geometric description and the equation for a conic section	G-GPE.3 (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.	Conic Sections - Forming Equations	G-GPE.3 Derive the equations of hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant	conic section, co-vertices, directrix, ellipse, focus, hyperbola, major axis, minor axis