Getting Ready to Teach Unit 6

Learning Path in the Common Core Standards

In previous units, the lessons focused on math concepts and skills as students applied these skills in word problems. In this unit, the focus is on solving problems as students use whole numbers, fractions, and decimals to find answers. Their work involves writing real world contexts and writing situation and solution equations for one-step, two-step, and multistep problems representing addition, subtraction, multiplication, and division.

Visual models and real world situations are used throughout the unit to represent problems and illustrate solutions.

***Math Expressions* VOCABULARY**

As you teach this unit, emphasize understanding of these terms:

- situation equation
- solution equation

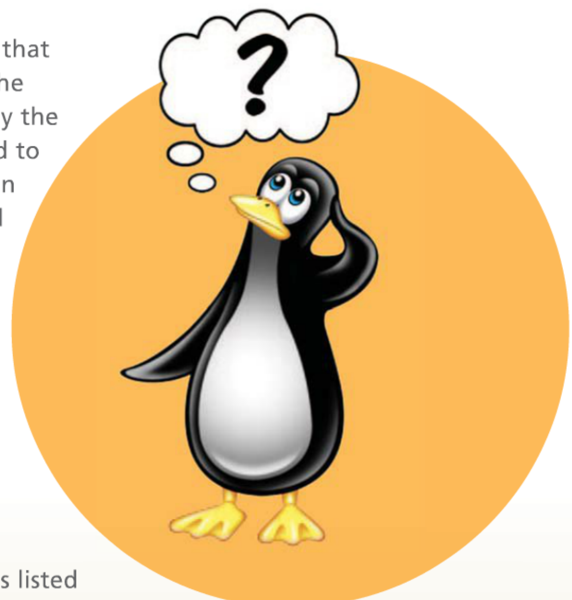
See the Teacher Glossary

Help Students Avoid Common Errors

Math Expressions gives students opportunities to analyze and correct errors, explaining why the reasoning was flawed.

In this unit, we use Puzzled Penguin to show typical errors that students make. Students enjoy teaching Puzzled Penguin the correct way, and explaining why this way is correct and why the error is wrong. The following common errors are presented to students as letters from Puzzled Penguin and as problems in the Teacher Edition that were solved incorrectly by Puzzled Penguin:

- **Lesson 1:** Writing incorrect situation and solution equations
- **Lesson 4:** Identifying an estimate that is not reasonable
- **Lesson 6:** Not choosing the correct operation
- **Lesson 10:** Using division to find a fractional part of a whole



In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.

The Problem Solving Process

Using the Mathematical Practices Throughout the program, *Math Expressions* integrates a research-based algebraic problem solving approach that focuses on problem types. Problem solving is a complex process that involves all eight of the CCSS Mathematical Processes. It is also an individual process that can vary considerably across students. Students may conceptualize, represent, and explain a given problem in different ways.

Mathematical Process	Student Actions
Understand the Problem Situation MP.1 Make sense of the problem. MP.2 Reason abstractly and quantitatively.	**Make Sense of the Language** Students use the problem language to conceptualize the real world situation.
Represent the Problem Situation MP.4 Model with mathematics. MP.7 Look for and make use of structure.	**Mathematize the Situation** Students focus on the mathematical aspects of the situation and make a math drawing and/or write a situation equation to represent the relationship of the numbers in the problem.
Solve the Problem MP.5 Use appropriate tools. MP.8 Use repeated reasoning.	**Find the Answer** Students use the math drawing and/or the situation/solution equation to find the unknown.
Check That the Answer Makes Sense MP.3 Critique the reasoning of others. MP.6 Attend to precision.	**Check the Answer in the Context of the Problem** Students write the answer to the problem, including a label. They explain and compare solutions with classmates.

Students are taught to make their own math drawings. Relating math drawings to equations helps them understand where in the drawing the total and the product are represented for each operation, and helps them solve equations with difficult unknowns.

Math Talk Learning Community

Research In the NSF research project that led to the development of *Math Expressions*, much work was done with helping teachers and students build learning communities within their classrooms. An important aspect of doing this is Math Talk. The researchers found three levels of Math Talk that go beyond the usual classroom routine of students simply solving problems and giving answers, and the teacher asking questions and offering explanations. It is expected that at Grade 5, students will engage in Math Talk at all levels.

Math Talk Level 1 A student briefly explains his or her thinking to others. The teacher helps students listen to and help others, models fuller explaining and questioning by others, and briefly probes and extends students' ideas.

Example Word Problem

> Anja has worked at her job for $6\frac{1}{2}$ years. Each year she works 48 weeks, and each week she works $37\frac{1}{2}$ hours. How many hours does Anja work each year?

What information *is not needed* to solve this problem?

Trevor: $6\frac{1}{2}$ years—the length of time Anja has worked at her job.

What information *is needed* to solve the problem?

Trevor: She works 48 weeks each year and $37\frac{1}{2}$ hours each week.

What would you do to find the number of hours?

Gabrielle: Multiplying $37\frac{1}{2}$ by 48 will tell us the number of hours she works each year. 48 times $37\frac{1}{2}$ is 1,800.

$$48 \times 37\frac{1}{2} = \frac{48}{1} \times \frac{75}{2} = \frac{3{,}600}{2} = 1{,}800$$

How would you estimate to check the answer?

Anthony: I would use rounding; $37\frac{1}{2}$ rounds to 40 and 48 rounds to 50. The exact answer should be close to 40×50, which is 2,000.

Math Talk Level 2 A student gives a fuller explanation and answers questions from other students. The teacher helps students listen to and ask good questions, models full explaining and questioning (especially for new topics), and probes more deeply to help students compare and contrast methods.

Example Word Problem

A shopping mall has 215 rows of parking spaces with 35 spaces in each row. A special permit is required to park in the first 6 spaces of 75 rows. How many parking spaces (s) do not require a special permit?

How can you restate this problem in different ways?

Jordan: A parking lot has 215 parking spaces. Some of the spaces require a permit. Some of the spaces don't.

Maria: The total number of parking spaces is made up of two parts, and one part—the number of spaces that do not require a special permit—is unknown.

How can we find the total number of parking spaces?

Destiny: I would multiply the number of rows by the number in each row. So I would multiply 215 by 35.

Miguel: 35 times 215 is 7,525. But we also need to find how many special permit spaces there are.

$$\begin{array}{r} 215 \\ \times\ 35 \\ \hline 1075 \\ +\ 6450 \\ \hline 7{,}525 \end{array}$$

Jasmine: We could find that number if we multiply 6 times 75. That number is 450.

$$\begin{array}{r} 75 \\ \times\ 6 \\ \hline 450 \end{array}$$

So how do we find the number of parking spaces that do not require a special permit?

Jada: We have to subtract 450 from 7,525 which is 7,075.

$$\begin{array}{r} 7{,}525 \\ -\ 450 \\ \hline 7{,}075 \end{array}$$

Austin: So 7,075 parking spaces do not require a special permit.

Math Talk Level 3 The explaining student manages the questioning and justifying. Students assist each other in understanding and correcting errors and in explaining more fully. The teacher monitors and assists and extends only as needed.

Example Word Problem

> The charge for an automobile repair was $328.50 for parts and $64 per hour for labor. The repair took $3\frac{3}{4}$ hours. What was the total cost (c) of the repair?

Kai, explain how to solve the problem.

Kai: We want to find the total cost of the repair. So I would multiply $64 times $\frac{15}{4}$ and add the product to $328.50.

$$\frac{64}{1} \times \frac{15}{4} = \frac{960}{4} = 240$$

$$\begin{array}{r} \$328.50 \\ +\ 240.00 \\ \hline \$568.50 \end{array}$$

Larissa: Why did you multiply $64 by $\frac{15}{4}$?

Kai: Because $3\frac{3}{4}$ is the same as $\frac{15}{4}$. I'm finding the cost of labor.

Muriel: Couldn't we have multiplied $64 by 3 and $64 by $\frac{3}{4}$ and then added the products together?

Can we use both ways to solve the problem?

Hector: 3 times $64 is $192 and $\frac{3}{4}$ times $64 is $48, and the total of those two amounts is $240.

$$\begin{array}{r} 64 \\ \times\ 3 \\ \hline 192 \end{array}$$

$$\frac{3}{4} \times \frac{64}{1} = \frac{192}{4} \text{ or } 48$$

Alexa: But we could have also multiplied $64 times 3.75 to find that same amount.

$$\begin{array}{r} 3.75 \\ \times\ \$64 \\ \hline 1500 \\ +\ 22500 \\ \hline \$240.00 \end{array}$$

Kai: Yes, we could have done that. But I multiplied $64 times $\frac{15}{4}$ and also found that the cost of labor was $240. Then I added $240 to $328.50 for a total of $568.50.

Did everyone get the same answer to this problem?

Class: Yes.

Summary Math Talk is important not only for discussing solutions to word problems, but also for any kind of mathematical thinking students do, such as simplifying an expression or solving an equation.

Writing Equations to Solve Problems

Lessons **1** **2**

Situation and Solution Equations for Addition and Subtraction

Throughout the lessons in Unit 6, students write one or two equations to solve word problems. Writing two equations involves writing a situation equation and a solution equation.

A *situation equation* shows the structure or relationship of the information in a problem. Students who write a situation equation to represent a problem use their understanding of inverse operations to then rewrite the equation as a solution equation.

A *solution equation* shows the operation that is used to solve a problem. If only one equation is written by students and used to solve a problem, the equation is a solution equation.

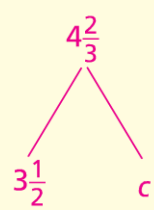
Model the Math Students work within addition and subtraction contexts and represent these problems with a break apart drawing. In a break apart drawing for addition and subtraction, the total is placed at the top. For example, the break apart drawing below models four related addition and subtraction equations.

$2.4 + 7.7 = 10.1$ $\quad$ $10.1 - 7.7 = 2.4$

$7.7 + 2.4 = 10.1$ $\quad$ $10.1 - 2.4 = 7.7$

Students model decimal and fraction problems using break apart drawings and write situation and solution equations to solve the problems.

> Jalen had $4\frac{2}{3}$ pounds of modeling clay and used $3\frac{1}{2}$ pounds for a craft project. How many pounds of clay were not used?



from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

An Algebraic Perspective Students begin developing an algebraic perspective many years before they will use formal algebraic symbols and methods. They read to understand the problem situation, represent the situation and its quantitative relationships with expressions and equations, and then manipulate that representation if necessary, using properties of operations and/or relationships between operations. Linking equations to concrete materials, drawings, and other representations of problem situations affords deep and flexible understandings of these building blocks of algebra.

Problem Types for Addition and Subtraction At this grade level, students solve addition and subtraction problem types involving whole numbers, fractions, and decimals.

Put Together, Result Unknown

A $\frac{5}{8}$-inch thick paperback book is on top of a $\frac{15}{16}$-inch thick paperback book. What is the total thickness of books?

Put Together, Change Unknown

Enrique has two packages to mail. The weight of one package is $12\frac{1}{4}$ pounds. What is the weight of the second package if the total weight of the packages is $15\frac{1}{8}$ pounds?

Put Together, Start Unknown

A shopper spent $53.50 for a sweater and a T-shirt. What was the cost of the sweater if the cost of the T-shirt was $16.50?

Take Apart, Result Unknown

At a track and field meet, Cody's time in a sprint event was 17.6 seconds. What was Shaina's time if she completed the event in 1.08 fewer seconds?

Take Apart, Change Unknown

Altogether, 91,292 people live in Waterloo and Muscatine, two cities in Iowa. The population of Waterloo is 68,406 people. What is the population of Muscatine?

Take Apart, Start Unknown

Altogether, the fourth graders jumped 345,127 times. If the fifth graders had done 2,905 fewer jumps, there would have been a tie. How many jumps did the fifth graders do?

from **THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING**

Problem Types In Grades 3, 4, and 5, students extend their understandings of addition and subtraction problem types [...] to situations that involve fractions and decimals. Importantly, the situational meanings for addition and subtraction remain the same for fractions and decimals as for whole numbers.

Add To, Result Unknown

Walt is running to get in shape. He ran 1.5 miles the first day. On the second day, he ran 2.75 miles. How far did he run during the two days?

Add To, Change Unknown

Matt is competing in the long jump event. His first jump was 3.56 m. So far, the longest jump in the event is 4.02 m. How much farther must he jump to be in first place?

Add To, Start Unknown

Walt is running for exercise. He ran around Lake Blue and then ran 2.75 miles home. He ran for a total of 4.25 miles. How far did he run around Lake Blue?

Take From, Result Unknown

A bag contained $6\frac{2}{3}$ cups of flour. Scott used $2\frac{1}{3}$ cups to make some bread. How much flour was left in the bag after he made the bread?

Take From, Change Unknown

Maryl measured her younger brother on January 1 of this year and found that he was $40\frac{3}{16}$ inches tall. On January 1 of last year, he was $37\frac{15}{16}$ inches tall. How much did he grow in a year's time?

Take From, Start Unknown

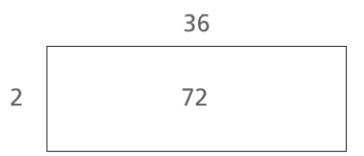
Sarita has some ribbon. After she used 23.8 cm of it, she had 50 cm left. How much ribbon did Sarita start with?

UNIT 6 | MATH BACKGROUND

Situation and Solution Equations for Multiplication and Division

Multiplication and division contexts are presented in Lesson 2. To help recognize and generate the situation and solution equations that represent the various contexts, students model the contexts using Rectangle Models. In a Rectangle Model, the product (or dividend) is placed inside the rectangle.

Model the Math The Rectangle Model below represents four related equations.



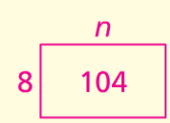
$2 \times 36 = 72$ $72 \div 36 = 2$

$36 \times 2 = 72$ $72 \div 2 = 36$

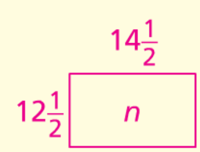
A Rectangle Model helps students see the information that is unknown and recognize the operation that is used to find that information. For example, when a completed rectangle shows a product in the middle and a factor as the length or width, students recognize division as the operation that is used to find a missing factor.

Inverse Operations Students build upon their understanding of inverse operations by using Rectangle Models as springboards for writing the situation and solution equations that are used to solve multiplication and division problems involving whole numbers, fractions, and decimals.

The musicians in a marching band are arranged in equal rows, with 8 musicians in each row. Altogether, the band has 104 musicians. In how many rows are the musicians marching?



Elena has chosen carpet that costs $4.55 per square foot for a rectangular floor that measures $12\frac{1}{2}$ feet by $14\frac{1}{2}$ feet. How many square feet of carpet is needed to cover the floor?



Problem Types for Multiplication and Division Problems The following multiplication and division problem types appear throughout Grade 5.

Equal Groups, Product Unknown

On the first day of soccer practice, $\frac{2}{5}$ of the players were wearing new shoes. The team has 20 players. How many players were wearing new shoes?

Equal Groups, Group Size Unknown

To get ready for her first semester of school, Jayna spent a total of $7.92 for eight identical notebooks. What was the cost of each notebook?

Equal Groups, Number of Groups Unknown

How many individual pieces of cheese, each weighing $\frac{1}{4}$ lb, can be cut from a block of cheese weighing 5 pounds?

Arrays, Product Unknown

A theater has 39 rows of seats. Each row has 54 seats. How many seats are in the theater?

Arrays, Group Size Unknown

In a school gymnasium, 588 students were seated for an assembly in 21 equal rows. What number of students were seated in each row?

Arrays, Number of Groups Unknown

The musicians in a marching band are arranged in equal rows, with 8 musicians in each row. Altogether, the band has 104 musicians. In how many rows are the musicians marching?

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Connections Students extend their whole number work with adding and subtracting and multiplying and dividing situations to decimal numbers and fractions. Each of these extensions can begin with problems that include all of the subtypes of [problem] situations. The operations of addition, subtraction, multiplication, and division continue to be used in the same way in these problem situations when they are extended to fractions and decimals (although making these extensions is not automatic or easy for all students).

Multiplication and Division Problem Types (continued)

Area, Product Unknown

Elena has chosen carpet that costs \$4.55 per square foot for a rectangular floor that measures $12\frac{1}{2}$ feet by $14\frac{1}{2}$ feet. How many square feet of carpet is needed to cover the floor?

Area, Side Unknown

A sidewalk covers 3,372 square feet. If the sidewalk is 4 feet wide, what is its length?

Compare, Product Unknown (Measurement Example)

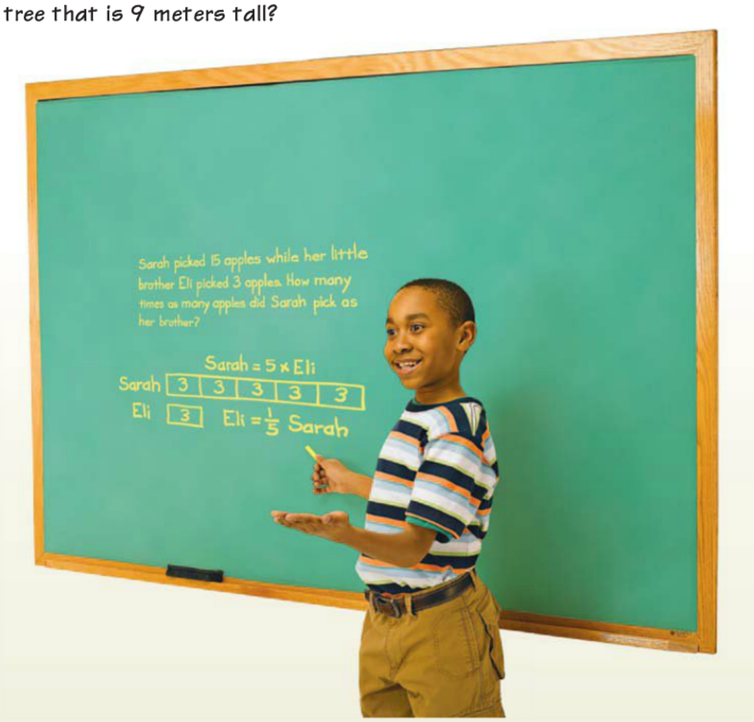
The length of an unstretched spring is 120 cm. How long (l) will the spring be if it is stretched to 3 times that length?

Compare, Group Size Unknown (Measurement Example)

The length of a collapsed fishing pole (c) is $\frac{1}{8}$ times as long as its extended length. The extended length of the pole is 16 feet. What is its collapsed length?

Compare, Number of Groups Unknown (Measurement Example)

A maple tree in the backyard of a home has a height of 0.75 meters. How many times as tall (t) is a nearby hickory tree that is 9 meters tall?

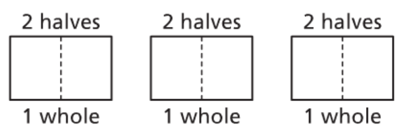


Write Multiplication and Division Word Problems

Lesson 3

Fractions and Decimals A variety of multiplication and division equations involving fractions and decimals are presented in Lesson 3. For each equation, students draw a model to represent the product or quotient and then, using words, write a problem for which the equation is sensible.

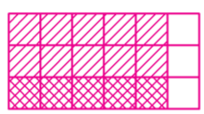
Model the Math Students may infer that the division equation $3 \div \frac{1}{2} = 6$ represents the idea of cutting 3 whole sheets of paper into halves, creating 6 halves. A possible model that may be drawn for this context and equation is shown below.



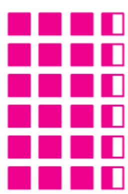
Other equations that are presented in Lesson 3, and some possible models for those equations, are shown below.

$\frac{3}{4} \cdot 2 = \frac{6}{4}$

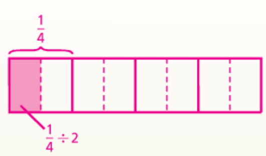
$\frac{5}{6} \cdot \frac{1}{3} = \frac{5}{18}$



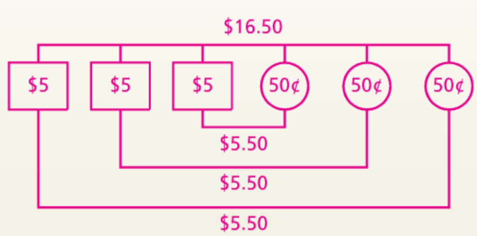
$\$6 \cdot 3.5 = \$21$



$\frac{1}{4} \div 2 = \frac{1}{8}$



$\$16.50 \div 3 = \$5.50$



from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS—FRACTIONS

Writing Problems In Grade 5, [students] connect fractions with division, understanding that

$$5 \div 3 = \frac{5}{3},$$

or, more generally, $\frac{a}{b} = a \div b$ for whole numbers a and b, with b not equal to zero. They can explain this by working with their understanding of division as equal sharing. They also create story contexts to represent problems involving division of whole numbers.

Determine Reasonableness

Lessons 1 2 4

Addition and Subtraction An important part of performing computations and solving equations is checking exact answers for reasonableness. Lesson 1 provides opportunities for students to develop number sense and reasoning skills.

Suppose you were asked to add the decimals at the right, and you wrote 2.07 as your answer. Without using pencil and paper to actually add the decimals, give a reason why an answer of 2.07 is not reasonable.

$$\begin{array}{r} 2.65 \\ +\ 0.42 \\ \hline \end{array}$$

Suppose you were asked to subtract the fractions at the right, and you wrote $\frac{5}{6}$ as your answer. Without using pencil and paper to actually subtract the fractions, give a reason why an answer of $\frac{5}{6}$ is not reasonable.

$$\frac{1}{2} - \frac{1}{3}$$

Multiplication and Division Lesson 2 also provides opportunities for students to develop number sense and reasoning skills.

Suppose you were asked to multiply the numbers at the right, and you wrote 15,000 as your answer. Without using pencil and paper to actually multiply the numbers, give a reason why an answer of 15,000 is not reasonable.

$$2{,}500 \times 0.6$$

Suppose you were asked to divide the numbers at the right, and you wrote 30 as your answer. Without using pencil and paper to actually divide the numbers, give a reason why an answer of 30 is not reasonable.

$$90 \div \frac{1}{3}$$

Using Number Sense Although students can check exact answers by performing computations a second time, a more efficient method involves using number sense to gain a general expectation of what exact answers should be. In Lesson 4, students are asked to use number sense and a variety of strategies to help decide the reasonableness of their exact answers.

Using Rounding One such strategy involves rounding. Students determine reasonableness by comparing an exact difference to a rounded difference.

> Altogether, 91,292 people live in Waterloo and Muscatine, two cities in Iowa. The population of Waterloo is 68,406 people. What is the population of Muscatine?

Using Estimation and Mental Math Estimation and mental math are also components of number sense, and used by students to determine the reasonableness of a computed quotient.

> In a school gymnasium, 588 students were seated for an assembly in 21 equal rows. What number of students were seated in each row?

Using Basic Facts Another number sense strategy involves using a pattern of basic facts to predict two consecutive whole numbers the exact answer should be between.

> A $45 award will be shared equally by 6 friends. In dollars, what is each friend's share of the award?

Using Benchmarks It is especially important for students to check computations involving fractions because generally speaking, many students are less comfortable working with fractions than with decimal or whole numbers, and less comfort often translates to more errors.

A way for students to check their computations involving fractions is to use benchmark values. Whole numbers and halves are examples of benchmark values.

> A $\frac{5}{8}$-inch thick paperback book is on top of a $\frac{15}{16}$-inch thick paperback book. What is the total thickness of books?

Summary All of the strategies in Lesson 4 involve number sense, and the goal of each strategy is for students to gain a sense of what to expect for an exact answer without having to check for reasonableness by performing computations a second time.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS—FRACTIONS

Reasonable Answers Students make sense of fractional quantities when solving word problems, estimating answers mentally to see if they make sense.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS—FRACTIONS

Using Benchmarks Students also reason using benchmarks such as $\frac{1}{2}$ and 1. For example, they see that $\frac{7}{8}$ is less than $\frac{13}{12}$ because $\frac{7}{8}$ is less than 1 (and is therefore to the left of 1 [on a number line]) but $\frac{13}{12}$ is greater than 1 (and is therefore to the right of 1).

Comparison Word Problems

Lessons 5 6 7

Language of Comparison Problems Students work with the language of comparison word problems in Lesson 5, and are presented with language that is *leading* and language that is *misleading*.

Leading Language The word *more* in the problem below suggests addition, and addition is used to solve the problem.

> Altogether, the Blue Team jumped 11,485 times. The Red Team did 827 more jumps than the Blue Team. How many jumps did the Red Team do?

The phrase *times as many* in the situation below suggests multiplication, and multiplication is used to solve the problem.

> Maria scored 6 points in the basketball game. Suzanne scored 4 times as many points as Maria. How many points did Suzanne score?

Misleading Language Although the word *more* in the situation below suggests addition, subtraction is the operation used to solve the problem.

> Ted jumped 1,300 times. He did 100 more jumps than Mario. How many jumps did Mario do?

In contrast to the problem about Maria above, division is the operation used to solve other *times as many* problems.

> Mr. Wagner has 32 horses on his farm. He has 4 times as many horses as Mr. Cruz. How many horses does Mr. Cruz have?

Summary The goal for students completing Lesson 5 is to understand that the language of a problem must be considered in the whole context of the problem because the meaning of the language may be different in context than by itself.

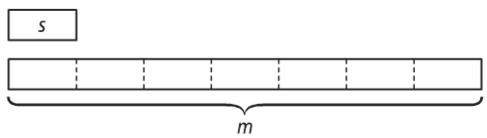
from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Comparison Problems In an additive comparison, the underlying question is *what amount would be added to one quantity* in order to result in the other. In a multiplicative comparison, the underlying question is *what factor would multiply one quantity* in order to result in the other.

Multiplicative Comparison Problems Relating math drawings to equations helps students understand where the total and the product are for each operation. This is especially true for multiplicative comparison problems that involve misleading language. Drawing comparison bars helps students show which quantity is larger, and helps them identify a solution equation and computation that can be used to solve the problem.

Writing Comparison Equations Students learn in Lesson 6 that both multiplication and division can be used to compare two quantities.

The comparison bars below represent the time a student worked on spelling (*s*) and math (*m*) homework.



1. Write a comparison sentence that includes the words "as long as" and compares
   a. *m* to *s*. *m* is seven times as long as *s*
   b. *s* to *m*. *s* is one-seventh as long as *m*
2. Write a comparison equation that compares
   a. *m* to *s*. $m = 7 \cdot s$ or $m = 7s$
   b. *s* to *m*. $s = \frac{1}{7} \cdot m$ or $s = \frac{1}{7}m$
3. Write a division equation that compares *s* to *m*. $s = m \div 7$ or $s = \frac{m}{7}$

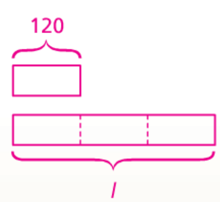
from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Multiplicative Comparison Problems In Grade 5, unit fractions language such as "one third as much" may be used. Multiplying and unit fraction language change the subject of the comparing sentence, e.g., "A red hat costs *A* times as much as the blue hat" results in the same comparison as "A blue hat costs $\frac{1}{A}$ times as much as the red hat," but has a different subject.

Multiplicative Comparison Problem Types

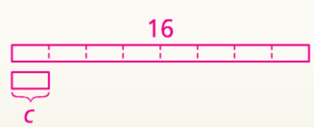
Compare, Product Unknown (Measurement Example)

The length of an unstretched spring is 120 cm. How long (*l*) will the spring be if it is stretched to 3 times that length?



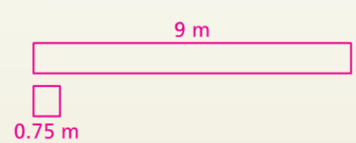
Compare, Group Size Unknown (Measurement Example)

The length of a collapsed fishing pole (*c*) is $\frac{1}{8}$ times as long as its extended length. The extended length of the pole is 16 feet. What is its collapsed length?



Compare, Number of Groups Unknown (Measurement Example)

A maple tree in the backyard of a home has a height of 0.75 meters. How many times as tall (*t*) is a nearby hickory tree that is 9 meters tall?



UNIT 6 MATH BACKGROUND

Multiplicative Comparison Problem Types (continued)

Compare, Product Unknown (Equal Groups Example)

At Southtown High School, the number of students in band is $1\frac{3}{4}$ times the number in orchestra. If 56 students are in orchestra, how many are in band?

Compare, Group Size Unknown (Equal Groups Example)

Dana has 9 CDs. She has $\frac{1}{5}$ as many as Sonya. How many CDs does Sonya have?

Compare, Number of Groups Unknown (Equal Groups Example)

To prepare for a test, Esmerelda studied 0.8 times as long as Mallory. Mallory studied for 50 minutes. How long (l) did Esmerelda study?

Multiplication and Scaling Students are also introduced to the idea of scaling in Lesson 6. Scaling is a number sense concept, and involves predicting the effect of resizing one factor of a multiplication.

> Gina and Mario each receive a weekly allowance. So far this year, Gina has saved $20 and Mario has saved 0.4 times that amount. Who has saved the greatest amount of money?

> Last week Camila worked 40 hours. Sergio worked $\frac{4}{5}$ that length of time. Which person worked more hours last week?

> On a math quiz, Juan was asked to find these two products:
>
> $3 \times 10.6 \qquad 2.7 \times 10.6$
>
> **a.** Without using pencil and paper to actually find the products, how will the product of 3×10.6 compare to the product of 2.7×10.6?
>
> **b.** How will the product of 2.7×10.6 compare to the product of 3×10.6?

Along with rounding, estimation, mental math, and benchmarks, scaling is another strategy students can use to help decide the reasonableness of an exact answer.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Scaling As students' notions of quantity evolve and generalize from discrete to continuous during Grades 3–5, their notions of multiplication evolves and generalizes. This evolution deserves special attention because it begins in [Operations and Algebraic Thinking] but ends in [Numbers and Operations—Fractions]. Thus, the concept of multiplication begins in Grade 3 with an entirely discrete notion of "equal groups." By Grade 4, students can also interpret a multiplication equation as a statement of comparison involving the notion "times as much." This notion has more affinity to continuous quantities, e.g., $3 = 4 \times \frac{3}{4}$ might describe how 3 cups of flour are 4 times as much as $\frac{3}{4}$ cup of flour. By Grade 5, when students multiply fractions in general, products can be larger or smaller than either factor, and multiplication can be seen as an operation that "stretches or shrinks" by a scale factor. This view of multiplication as scaling is the appropriate notion for reasoning multiplicatively with continuous quantities.

Identify Comparison Problem Types Students may conceptualize, represent, and explain a given problem in different ways. Drawings help students show which quantity is larger, and identify a solution equation and computation that can be used to solve the problem. Rather than use the subtraction equation, students often prefer to represent an addition equation as Smaller + Difference = Larger as the only representation for comparison situations.

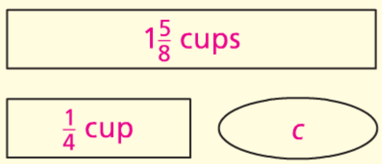
Writing Equations Students identify and solve real world one-step additive and multiplicative comparison problems in Lesson 7.

Newborn baby Lila is 44.5 centimeters tall. Her older brother Tremaine is 4 times as tall. How tall (t) is Tremaine?

Type of comparison: multiplicative

Equation and answer: $44.5 \bullet 4 = t$; $t = 178$ cm tall

Brandon has $\frac{1}{4}$ cup of flour, and would like to make a recipe that requires $1\frac{5}{8}$ cups of flour. How many more cups (c) of flour are needed for the recipe?



Type of comparison: additive

Equation and answer: $\frac{1}{4} + c = 1\frac{5}{8}$; $c = 1\frac{3}{8}$ more cups

Additive Comparison Problem Types

Compare, Difference Unknown

Julia jumped rope 1,200 times. Samantha jumped 1,100 times. How many more jumps did Julia do?

Compare, Bigger Unknown

Altogether, the Blue Team jumped 11,485 times. The Red Team did 827 more jumps than the Blue Team. How many jumps did the Red Team do?

Compare, Smaller Unknown

Ted jumped 1,300 times. He did 100 more jumps than Mario. How many jumps did Mario do?

***from* THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING**

Additive Comparison Problems

Compare situations can be represented with tape diagrams showing the compared quantities (one smaller and one larger) and the difference. Other diagrams showing two numbers and the unknown can also be used. Such diagrams are a major step forward because the same diagrams can represent the adding and subtracting situations for all of the kinds of numbers students encounter in later grades (multi-digit whole numbers, fractions, decimals, variables).

Problems With More Than One Step

Lessons 8 9 10

Different Approaches for Solving Problems There are multiple entry points to solving two-step or multistep problems. In other words, there are no algorithmic one-way approaches for solving those problems. Encouraging individual approaches to solve problems may lead to new solutions to new problems not yet imagined. For a given problem, some students may use a forward approach, immediately finding an answer for a first part of a problem and then deciding if that answer is a needed step toward the overall solution. Others may work backwards, using two or more drawings, and/or writing two or more equations.

All of the problems in Lessons 1–7 involved one step. The problems in Lessons 8–10 involve more than one step. Problems involving more than one step typically involve more than one operation, and often the order in which those operations are performed is important.

Two-Step Problems In Lesson 8, students learn that parentheses in an expression or equation represent grouping symbols that indicate which operation should be performed first. Students begin their work by writing equations to represent problems that are solved using two steps, and include parentheses to indicate which operation should be performed first. The two-step problems include whole numbers, decimals, and fractions.

> A recipe that makes 6 servings requires $1\frac{1}{4}$ cups of flour. How much flour (f) would be needed to make the recipe for one-half the number of servings?

> An apple orchard in Minnesota has 8 rows of 26 honeycrisp trees and 14 rows of 23 red delicious trees. How many honeycrisp and red delicious trees (t) are in the orchard?

> An investor purchased 250 shares of stock. Calculate the investor's total cost (c) if the price per share was \$18.40 and a fee of \$65.75 was charged for the transaction.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Two-Step and Multistep Problems
Students use the new kinds of numbers, fractions and decimals, in geometric measurement and data problems and extend to some two-step and multi-step problems involving all four operations. In order to keep the difficulty level from becoming extreme, there should be a tradeoff between the algebraic or situational complexity of any given problem and its computational difficulty taking into account the kinds of numbers involved.

Multistep Problems Although the problems in Lesson 9 again represent whole numbers, decimals, and fractions in real world contexts, the problems differ in that they are multistep. In multistep problems, more than two steps must be completed to generate the solutions.

> An investor purchased 150 shares of stock at $13.60 per share and sold the shares later for $11.92 per share. Calculate the profit or loss of the transaction.
>
> **a.** What equation can be used to find the amount of money needed to buy (*b*) the shares?
>
> **b.** What equation can be used to find the amount of money received for selling (*s*) the shares?
>
> **c.** Does the transaction represent a profit or loss? Why?
>
> **d.** What equation can be used to calculate the loss (*l*)? Solve your equation to calculate the loss.

> This week an employee is scheduled to work $7\frac{1}{2}$ hours each day, Monday through Friday, and 2 hours on Saturday morning. If the employee's goal is to work 40 hours, how many additional hours (*h*) must be worked?

Other Types of Problems Included in this unit are problems that involve too much information, too little information, and interpreting remainders. Problems involving hidden information are taught in Unit 8.

Too Much Information

> A wallpaper border is being pasted on the walls of a rectangular room that measures 12 feet by $14\frac{1}{2}$ feet. The cost of the border is $6.50 per foot. How many feet of border is needed for the room?

Too Little Information

> Ms. Bleyleven has 11 windows in her house. The heights in centimeters of 4 windows are shown below.
>
> 160.2 cm   163 cm   155.9 cm   158.5 cm
>
> How many windows in her house have a height that is a whole number of centimeters?

Interpret Remainders Lesson 10 is a practice lesson that includes all of the skills and strategies students learned in this unit. In addition, students interpret division remainders.

> At closing time, 55 adults and 89 students are waiting in line to ride an amusement park roller coaster. The capacity of the coaster is 38 riders. How many trips (*t*) must the coaster make to give all of the people in line a ride? How many people will be on the last trip of the day?

> Six teachers, seventy-eight students, and ten parents are boarding buses for a school field trip. Each bus can carry 32 passengers. If the passengers board each bus until it is full, how many passengers (*p*) will be on the bus that is not full?

Focus on Mathematical Practices

Lesson 11

The Standards for Mathematical Practice are included in every lesson of this unit. However, there is an additional lesson that focuses on all eight Mathematical Practices. In this lesson, students use what they know about multiplying whole numbers and decimals to complete computations related to gymnastics and diving scores.