Getting Ready to Teach Unit 1

Learning Path in the Common Core Standards

In this unit, students study fractions and mixed numbers. They find equivalent fractions, compare fractions, and add and subtract fractions and mixed numbers.

Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent problems with like denominators. They make reasonable estimates of the sums and differences.

Visual models and real world situations are used throughout the unit to illustrate important fraction concepts.

***Math Expressions* VOCABULARY**

As you teach this unit, emphasize understanding of these terms:

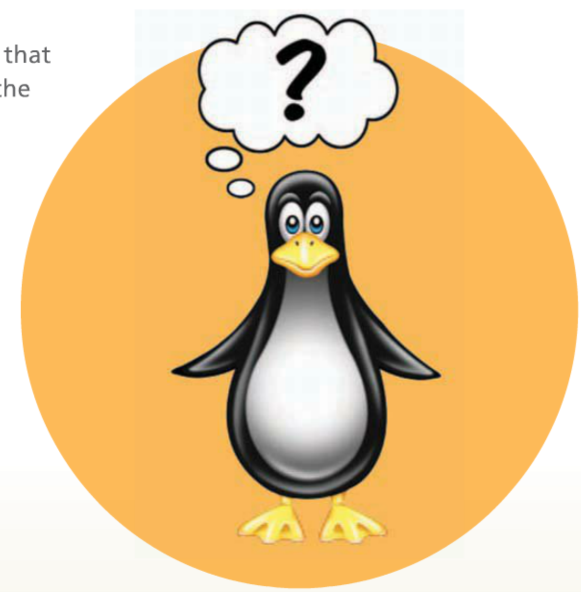
- unsimplify
- *n*-split

See the Teacher Glossary

Help Students Avoid Common Errors

Math Expressions gives students opportunities to analyze and correct errors, explaining why the reasoning was flawed.

In this unit, we use Puzzled Penguin to show typical errors that students make. Students enjoy teaching Puzzled Penguin the correct way, and explaining why this way is correct and why the error is wrong. The following common errors are presented to students as letters from Puzzled Penguin and as problems in the Teacher Edition that were solved incorrectly by Puzzled Penguin.



- **Lesson 3:** Generating an equivalent fraction by multiplying only one part of the fraction
- **Lesson 5:** Interpreting a mixed number as a whole number times a fraction
- **Lesson 6:** When ungrouping a mixed number to subtract, forgetting to reduce the whole number part by 1
- **Lesson 7:** Adding fractions by adding numerators and adding denominators
- **Lesson 9:** Adding mixed numbers by adding numerators and adding denominators; when subtracting mixed numbers, subtracting the lesser fraction from the greater fraction, even though the lesser fraction is in the minuend
- **Lesson 11:** Not checking answers for reasonableness

In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As a part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.

Build with Unit Fractions

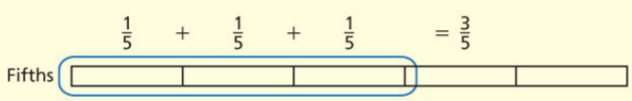
Lesson **1**

Unit Fractions The fraction bars on the MathBoard are used to discuss unit fractions. A unit fraction has the form $\frac{1}{n}$, where n is the number of equal parts the whole is divided into. Students recall that a unit fraction represents one of those parts of a whole. For example, the unit fraction $\frac{1}{5}$ represents one of five equal parts of a whole.

The MathBoard fraction bars allow students to observe that unit fractions with greater denominators represent smaller parts of a whole. For example, $\frac{1}{5}$ is smaller than $\frac{1}{3}$. Students can reason that this makes sense because the more parts the same whole is divided into, the smaller each part must be.

Non-Unit Fractions Fraction bars allow students to see how a whole and other non-unit fractions are built by combining (adding) unit fractions.

- Any fraction that is not a unit fraction can be built by adding unit fractions. For example, $\frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$. This can also be expressed as $\frac{3}{5} = 3 \times \frac{1}{5}$.



- The denominator of a fraction tells the number of unit fractions in the whole. For example, for $\frac{3}{5}$, the whole is made up of five unit fractions (specifically, five fifths, or five $\frac{1}{5}$s).
- The numerator of a fraction tells the number of unit fractions in the fraction. For example, the fraction $\frac{3}{5}$ is made up of three unit fractions—in this case, three fifths, or three $\frac{1}{5}$s.

Viewing non-unit fractions as sums of unit fractions helps students avoid common errors in adding and subtracting fractions (including adding numerators and denominators, not just numerators).

$$\frac{3}{7} + \frac{2}{7} = \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} = \frac{5}{7}$$

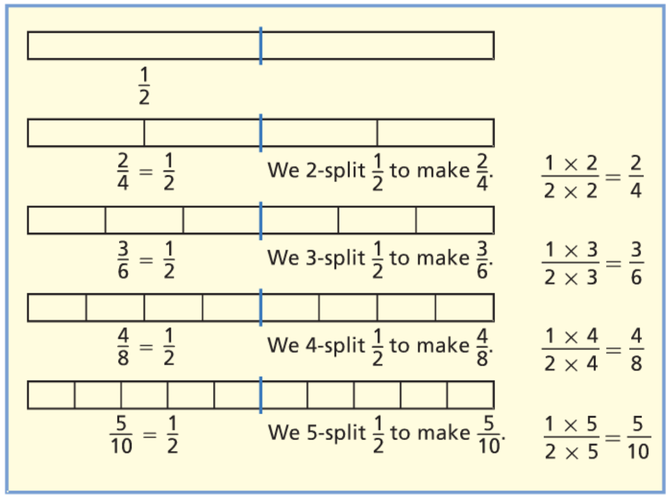
***from* THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS—FRACTIONS**

Unit Fractions The goal is for students to see unit fractions as the basic building blocks of fractions, in the same sense that the number 1 is the basic building block of the whole numbers; just as every whole number is obtained by combining a sufficient number of 1s, every fraction is obtained by combining a sufficient number of unit fractions.

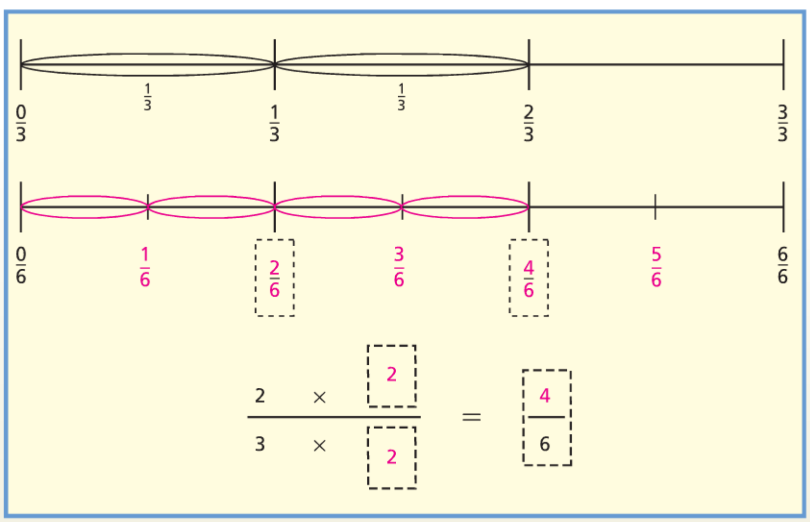
Equivalent Fractions

Lessons 2 3

Fraction Bars Students begin their work with equivalent fractions by looking at the portion of each fraction bar that is equivalent to $\frac{1}{2}$. They consider what we must do both to the fraction-bar model for $\frac{1}{2}$ and to the numerical fraction $\frac{1}{2}$ to form equivalent fractions. For example, to go from the model for $\frac{1}{2}$ to the model for $\frac{3}{6}$, we must divide each half into three equal parts. (We say we must 3-split $\frac{1}{2}$.) To get from the numerical fraction $\frac{1}{2}$ to $\frac{3}{6}$, we must multiply both the numerator and the denominator by 3.



Number Lines On a number line, students see that they can make equivalent fractions by dividing each interval into smaller intervals (that is, by dividing each unit fraction into smaller unit fractions). For example, by dividing each third into two equal intervals to make sixths, we can see that $\frac{2}{3}$ is equivalent to $\frac{4}{6}$. Dividing each interval in two parts is mathematically equivalent to multiplying both the numerator and denominator of $\frac{2}{3}$ by 2.



from the Progressions for the Common Core State Standards on Number and Operations—Fractions

Equivalent Fractions Students can use area models and number line diagrams to reason about equivalence. They see that the numerical process of multiplying the numerator and denominator of a fraction by the same number, *n,* corresponds physically to partitioning each unit fraction piece into *n* smaller equal pieces. The whole is then partitioned into *n* times as many pieces, and there are *n* times as many smaller unit fraction pieces as in the original fraction.

Students also make equivalent fractions on the number line by grouping intervals to make larger intervals (in other words, by grouping unit fractions to make greater unit fractions). For example, making groups of 5 fifteenths makes thirds. The fraction $\frac{10}{15}$ becomes the equivalent fraction $\frac{2}{3}$. This is equivalent to dividing the numerator and denominator by 5.

$$\frac{10 \div 5}{15 \div 5} = \frac{2}{3}$$

In *Math Expressions*, we use *simplify* to refer to the process of creating an equivalent fraction by dividing both parts of a fraction by the same number. We use *unsimplify* to refer to the process of creating an equivalent fraction by multiplying both parts by the same number. For example, we can simplify $\frac{9}{12}$ by dividing both parts by 3 to get $\frac{3}{4}$. We can unsimplify $\frac{2}{5}$ by multiplying both parts by 3 to get $\frac{6}{15}$.

Multiplication Table Lesson 3 solidifies the role of the multiplier in generating equivalent fractions. Students learn how equivalent fractions can be seen in the multiplication table. We can choose any two rows of the table. If we consider the numbers in one row to be numerators and the corresponding numbers in the other row to be denominators then we can make a chain of equivalent fractions.

The multipliers needed to make each fraction from the first fraction are in the top row of the table. For example, to get $\frac{18}{30}$ from $\frac{3}{5}$, we must use the multiplier 6, which is at the top of the column that contains both 18 and 30.

Using the multiplication table helps students answer questions like these: If I need a fraction equivalent to $\frac{4}{7}$ with a denominator of 56, what will the numerator be? If I need a fraction equivalent to $\frac{4}{7}$ with a numerator of 24, what will the denominator be?

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

$$\frac{3}{5} \quad \frac{6}{10} \quad \frac{9}{15} \quad \frac{12}{20} \quad \frac{15}{25} \quad \frac{18}{30} \quad \frac{21}{35} \quad \frac{24}{40} \quad \frac{27}{45} \quad \frac{30}{50}$$

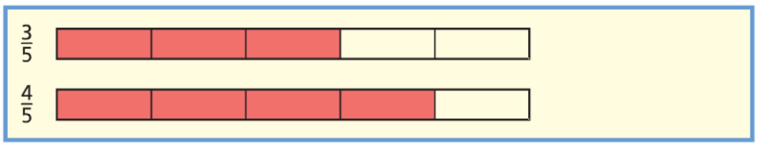
UNIT 1 MATH BACKGROUND

Compare Fractions

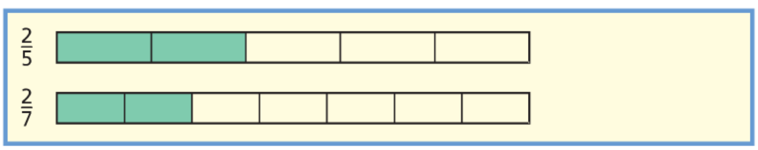
Lesson 4

In Lesson 4, students explore a variety of strategies for comparing fractions.

Like Denominators If two fractions have the same denominator, then they are made from the same-size unit fraction. The fraction with the greater numerator—that is, the fraction with the greater number of unit fractions—is visually larger and therefore the greater fraction.



Like Numerators If two fractions have the same numerator, then they are made from the same number of unit fractions. The fraction with the lesser denominator—that is, the fraction made from the greater unit fractions—is visually larger and therefore the greater fraction.



from the Progressions for the Common Core State Standards on Number and Operations—Fractions

Comparing fractions They see that for fractions that have the same denominator, the underlying unit fractions are the same size, so the fraction with the greater numerator is greater because it is made of more unit fractions. Students also see that for unit fractions, the one with the larger denominator is smaller, by reasoning, for example, that in order for more (identical) pieces to make the same whole, the pieces must be smaller. From this they reason that for fractions that have the same numerator, the fraction with the smaller denominator is greater.

Unlike Denominators To compare two fractions with different denominators, we can rewrite them as equivalent fractions with a common denominator. For example, to compare $\frac{5}{8}$ and $\frac{7}{12}$, we can use the common denominator 24.

$$\frac{5}{8} = \frac{5 \times 3}{8 \times 3} = \frac{15}{24} \qquad \frac{7}{12} = \frac{7 \times 2}{12 \times 2} = \frac{14}{24}$$

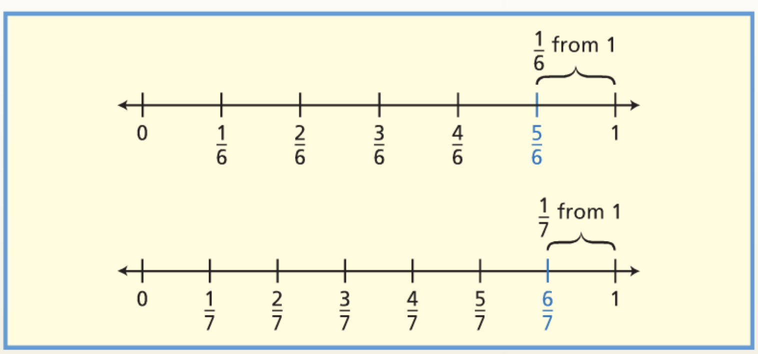
Because $\frac{15}{24} > \frac{14}{24}$, we know that $\frac{5}{8} > \frac{7}{12}$.

When finding a common denominator, students consider the following three cases; however, Lesson 4 emphasizes that we can *always* use the product of the denominators as the common denominator.

- *One denominator is a factor of the other*. In this case, we can use the greater denominator as the common denominator. For example, for $\frac{3}{5}$ and $\frac{7}{10}$, we can use 10 as the common denominator.
- *The denominators have no common factors except 1.* In this case, we can use the product of the denominators as the common denominator. For example, for $\frac{4}{5}$ and $\frac{5}{7}$, we can use 35 as the common denominator.
- *The denominators have a common factor greater than 1.* In this case, we can find a common denominator that is less than the product of the denominators. For example, for $\frac{5}{12}$ and $\frac{4}{9}$, we can use 36 as a common denominator.

Special Cases Students learn that in some special situations, we can compare fractions by using reasoning.

- If both fractions are close to $\frac{1}{2}$, we can compare both to $\frac{1}{2}$. If one fraction is greater than $\frac{1}{2}$ and the other is less than $\frac{1}{2}$, then the fraction greater than $\frac{1}{2}$ is greater. For example, because $\frac{5}{8} > \frac{1}{2}$ and $\frac{2}{5} < \frac{1}{2}$, we can conclude that $\frac{5}{8} > \frac{2}{5}$.
- If both fractions are close to 1 (and both are less than 1), we can compare the fractions to 1. The fraction closer to 1 is greater. For example, $\frac{5}{6}$ is $\frac{1}{6}$ away from 1. $\frac{6}{7}$ is $\frac{1}{7}$ away from 1. Because $\frac{1}{7} < \frac{1}{6}$, $\frac{6}{7}$ is closer to 1, so it is greater.



from the Progressions for the Common Core State Standards on Number and Operations—Fractions

Using Number Lines to Compare As students move towards thinking of fractions as points on the number line, they develop an understanding of order in terms of position. Given two fractions—thus two points on the number line— the one to the left is said to be smaller and the one to the right is said to be larger. This understanding of order as position will become important in Grade 6 when students start working with negative numbers.

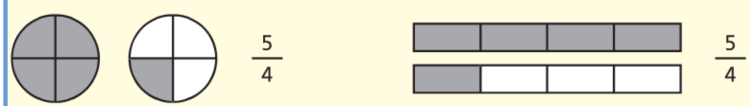
Fractions Greater Than 1 and Mixed Numbers

Lesson 5

Fractions Greater Than One The idea of building fractions from unit fractions is used to develop the ideas of fractions greater than 1 and mixed numbers. For example, the fraction $\frac{5}{4}$ is the sum of 5 fourths.

$$\frac{5}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$

Students use a variety of representations to represent $\frac{5}{4}$ and show that it is greater than one whole.



Students can group unit fractions to show that $\frac{5}{4}$ is one whole ($\frac{4}{4}$) and $\frac{1}{4}$ more.

$$\left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right) + \frac{1}{4} = 1\frac{1}{4}$$

Convert Between Forms Students use their own methods to convert between mixed numbers and fractions. Many students will use some or all of the methods below.

$$2\frac{3}{4} = 2 + \frac{3}{4} = 1 + 1 + \frac{3}{4} = \frac{4}{4} + \frac{4}{4} + \frac{3}{4} = \frac{11}{4}$$

Some students will see the shortcut of multiplying 2×4, to find that there are 8 fourths in 2, and then adding the 3 fourths.

To convert a fraction to a mixed number, students can reverse the thinking used above. Below is an example. Most students will not need to record all the steps.

$$\frac{17}{5} = \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{2}{5} = 1 + 1 + 1 + \frac{2}{5} = 3 + \frac{2}{5} = 3\frac{2}{5}$$

Add and Subtract Like Mixed Numbers

Lesson 6

In Lesson 6, students add and subtract mixed numbers with like denominators. To add, most students will use one of the following methods:

- Add whole number parts and fraction parts separately and regroup if needed.

Horizontally	Vertically
$1\frac{2}{3} + 1\frac{2}{3} = (1 + 1) + (\frac{2}{3} + \frac{2}{3})$ $= 2\frac{4}{3}$ $= 3\frac{1}{3}$	$\begin{array}{r} 1\frac{2}{3} \\ +1\frac{2}{3} \\ \hline 2\frac{4}{3} \end{array} = 3\frac{1}{3}$

- Rewrite the mixed numbers as fractions and add.

$$1\frac{2}{3} + 1\frac{2}{3} = \frac{5}{3} + \frac{5}{3} = \frac{10}{3} = 3\frac{1}{3}$$

To subtract, students are likely to use one of these methods:

- Subtract the whole number parts and fraction parts separately, ungrouping first if needed.

$$\begin{array}{r} 6\frac{6}{5} \\ \not{7}\frac{\not{1}}{5} \\ -\ 2\frac{4}{5} \\ \hline 4\frac{2}{5} \end{array}$$

- Add on from the lesser number to the greater number.

$2\frac{4}{5}$ to 3 to 7 to $7\frac{1}{5}$

$\frac{1}{5} + 4 + \frac{1}{5} = 4\frac{2}{5}$

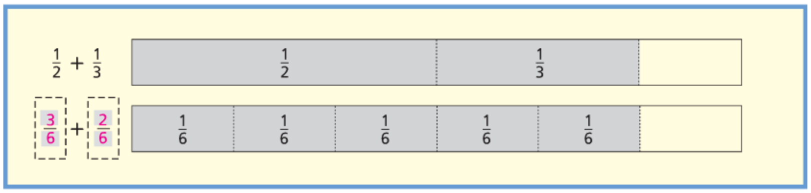
- Rewrite the mixed numbers as fractions and subtract.

$$7\frac{1}{5} - 2\frac{4}{5} = \frac{36}{5} - \frac{14}{5} = \frac{22}{5} = 4\frac{2}{5}$$

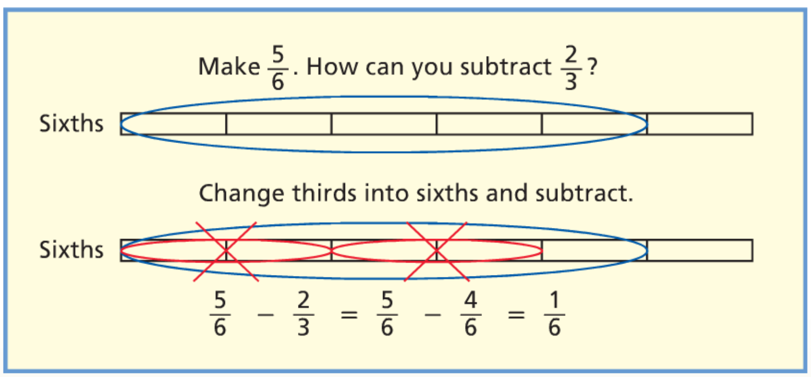
Add and Subtract Unlike Fractions and Mixed Numbers

Lessons 7 8 9 10

Add Unlike Fractions In Lesson 7, students add numbers with unlike denominators. Fraction bars are used to illustrate why we must rewrite the fractions with a common denominator before adding. For example, the top bar below shows $\frac{1}{2} + \frac{1}{3}$, but we can't tell what the total value is. The bottom bar shows that when we express each addend as a number of sixths, we can see that the sum is $\frac{5}{6}$.



Subtract Unlike Fractions Students subtract unlike fractions in Lesson 8. Fraction bars are again used to illustrate why it is necessary to rewrite the fractions with a common denominator.



Find Common Denominators Throughout Lessons 7 and 8, students use the strategies discussed in Lesson 4 to find common denominators. (See page 1EE for a list of these strategies.)

from the Progressions for the Common Core State Standards on Number and Operations—Fractions

Adding and Subtracting Fractions Students express both fractions in terms of a new denominator. For example, in calculating $\frac{2}{3} + \frac{5}{4}$, they reason that if each third in $\frac{2}{3}$ is subdivided into fourths, and if each fourth in $\frac{5}{4}$ is subdivided into thirds, then each fraction will be a sum of unit fractions with denominator $3 \times 4 = 4 \times 3 = 12$.

$$\frac{2}{3} + \frac{5}{4} = \frac{2 \times 4}{3 \times 4} + \frac{5 \times 3}{4 \times 3} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$$

In general two fractions can be added by subdividing the unit fractions in one using the denominator of the other.

Add and Subtract Unlike Mixed Numbers In Lessons 9 and 10, students add and subtract mixed numbers with unlike denominators. Such computations can be quite complex. They require finding equivalent fractions, adding or subtracting the whole numbers and fractions separately, and ungrouping if necessary for subtraction. Then two kinds of simplifying may be required: simplifying a fraction to the simplest form and changing a fraction greater than 1 to a mixed number, which must be added to the whole number. By sharing and discussing solution methods, students can develop effective and efficient strategies for doing these computations. The following discussion is from Lesson 9, page 69.

When presenting their solutions, pairs should describe each step, with other students asking questions or helping to clarify. Below is a discussion of Exercise 5.

Eric and Max, please explain how you solved Exercise 5.

Eric: First, we renamed the fractions so they had the same denominator. The denominators are 3 and 15. 3 is a factor of 15, so 15 is a common denominator. We renamed $\frac{1}{3}$ as $\frac{5}{15}$.

$$\begin{array}{rcl} 4\frac{1}{3} & = & 4\frac{5}{15} \\ -2\frac{7}{15} & = & 2\frac{7}{15} \\ \hline \end{array}$$

Max: We couldn't subract $\frac{7}{15}$ from $\frac{5}{15}$ because $\frac{5}{15}$ is smaller, so we had to ungroup. The 4 becomes a 3 because I'm giving a 1 to the fifteenths. One is the same as $\frac{15}{15}$. Adding $\frac{15}{15}$ to $\frac{5}{15}$, I get $\frac{20}{15}$.

$$\begin{array}{rcl} & & 3\frac{20}{15} \\ 4\frac{1}{3} & = & \not{4}\frac{\not{5}}{15} \\ -2\frac{7}{15} & = & 2\frac{7}{15} \\ \hline \end{array}$$

Eric: Then we just subtracted. We subtracted the whole numbers first: $3 - 2 = 1$. Then we subracted the fractions: $\frac{20}{15} - \frac{7}{15} = \frac{13}{15}$.

$$\begin{array}{rcl} & & 3\frac{20}{15} \\ 4\frac{1}{3} & = & \not{4}\frac{\not{5}}{15} \\ -2\frac{7}{15} & = & 2\frac{7}{15} \\ \hline & & 1\frac{13}{15} \end{array}$$

Estimation and Reasonable Answers

Lessons 11 12

In Lesson 11, students discuss strategies for mentally estimating sums and differences of fractions and mixed numbers and for determining whether answers are reasonable. These strategies include the following:

- *Using benchmarks of 0, $\frac{1}{2}$, and 1:* For example, consider $\frac{4}{9} + \frac{5}{6}$. Because $\frac{4}{9}$ is closer to $\frac{1}{2}$ than to 0 and $\frac{5}{6}$ is closer to 1 than to $\frac{1}{2}$, we know $\frac{4}{9} + \frac{5}{6}$ is close to $\frac{1}{2} + 1$, or $1\frac{1}{2}$.
- *Rounding to the nearest whole number:* For example, consider $2\frac{1}{3} + 5\frac{3}{4}$. Because $2\frac{1}{3}$ rounds to 2 and $5\frac{3}{4}$ rounds to 6, the sum should be about 8.
- *Using number sense and reasoning:* For example, consider the (incorrect) equation $\frac{5}{8} + \frac{1}{6} = \frac{6}{14}$. We can reason that because one of the addends, $\frac{5}{8}$, is more than $\frac{1}{2}$, it is impossible for the sum to be less than $\frac{1}{2}$. Since $\frac{6}{14} < \frac{1}{2}$, $\frac{6}{14}$ is not a reasonable answer.

In Lesson 12, students apply what they have learned throughout the unit to solve one-step and multistep word problems involving addition and subtraction of fractions and mixed numbers. They must also describe how they know their answer is reasonable.

> **4.** At a pizza party, the Mehta family ate $1\frac{3}{8}$ pizzas in all. They ate $\frac{9}{12}$ of a cheese pizza and some pepperoni pizza. How much pepperoni pizza did they eat?
>
> Equation and answer:
> $\frac{9}{12} + p = 1\frac{3}{8}$; $\frac{5}{8}$ pizzas
>
> Why is the answer reasonable?
> $\frac{9}{12}$ is close to 1 and $\frac{5}{8}$ is close to $\frac{1}{2}$, so the total should be close to $1\frac{1}{2}$.

Focus on Mathematical Practices

Lesson 13

The Standards for Mathematical Practice are included in every lesson of this unit. However, there is an additional lesson that focuses on all eight Mathematical Practices. In this lesson, students use what they know about adding and subtracting fractions to compute the dimensions of a bird hotel.

from the Progressions for the Common Core State Standards on Number and Operations—Fractions

estimating answers Students make sense of fractional quantities when solving word problems, estimating answers mentally to see if they make sense.