Getting Ready to Teach Unit 7

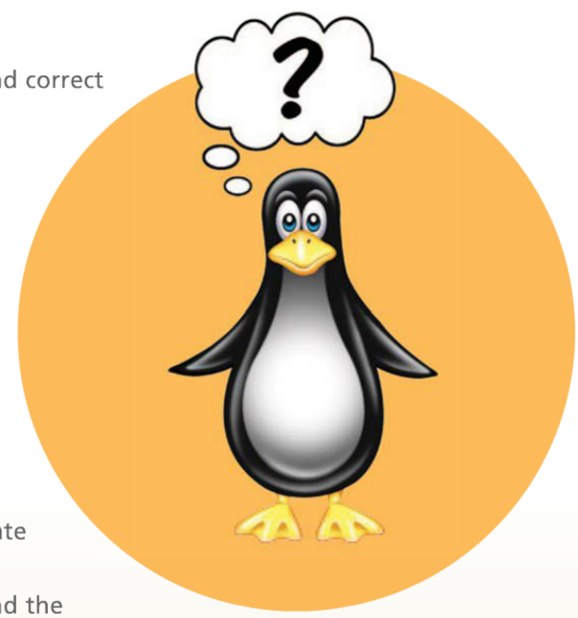
Learning Path in the Common Core Standards

In this unit, students build upon the concepts of fractions presented in Unit 6. The number line is presented as a powerful model to represent fractions. Students use their conceptual knowledge of fractions to develop procedures to compare fractions and to find equivalent fractions using common denominators.

Another goal of Unit 7 is to develop an understanding of decimal numbers by relating decimals to fractions and whole-number place values. Students are introduced to a variety of models for representing fractions and decimals as parts of a set (dimes and pennies), parts of a whole (decimal bar), and as a model of distance (number line). Students use the relationship between decimals and fractions to build decimal concepts, including decimal place value and comparing decimals less than and greater than 1.

Help Students Avoid Common Errors

Math Expressions gives students opportunities to analyze and correct errors, explaining why the reasoning was flawed.



In this unit, we use Puzzled Penguin to show typical errors that students make. Students enjoy explaining Puzzled Penguin's error and teaching Puzzled Penguin the correct way to compare fractions and relate fractions and decimals. The following common errors are presented to the students as letters from Puzzled Penguin and as problems in the Teacher Edition that were solved incorrectly by Puzzled Penguin.

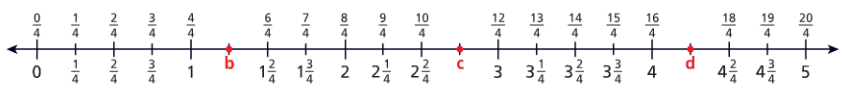
- **Lesson 1:** Incorrectly using denominators to compare fractions
- **Lesson 2:** Forgetting to consider whether an overestimate or an underestimate is more appropriate
- **Lesson 4:** Forgetting to multiply both the numerator and the denominator by the same number to write equivalent fractions
- **Lesson 6:** Forgetting to multiply the numerator and denominator in a fraction by the same number
- **Lesson 10:** When comparing decimals, not recognizing that the decimals being compared must refer to the same whole

In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As a part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.

Fraction Concepts

Lessons 1 2 3 4 5 6

Fractions on the Number Line This unit introduces the number line as another way to model fractions. Students compare the number line model with the fraction bar models they used in Unit 6. Students can see that the number of divisions between 0 and 1 determines the unit fraction. In the number line below the interval between 0 and 1 is divided into four equal parts, so each mark indicates $\frac{1}{4}$. The labels above the number line after 1 show numbers greater than 1. The labels below the line show the numbers as mixed numbers. Fractions greater than 1 and mixed numbers are different ways to represent the same distance on the number line.



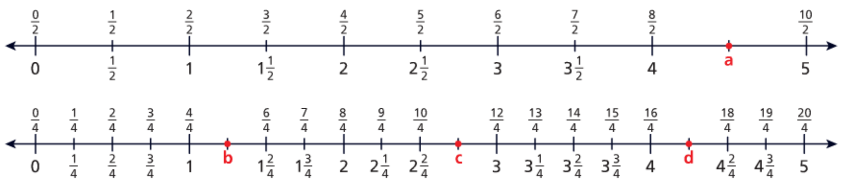
The number line can be used to help students find relationships between mixed numbers and fractions greater than 1. For example, by looking at the number line above, it is clear that $\frac{13}{4} = 3\frac{1}{4}$.

Students also use the number line to explore fraction benchmarks by estimating whether a fraction is closer to 0 or 1. It is beneficial for students to develop the ability to compare fractions to 0, $\frac{1}{2}$, and 1 because it will help them use mental math to compare fractions. For example, if students are comparing $\frac{3}{8}$ and $\frac{7}{12}$, and they are able to reason that if $\frac{3}{8}$ is less than $\frac{1}{2}$ and $\frac{7}{12}$ is greater than $\frac{1}{2}$, then $\frac{3}{8}$ is less than $\frac{7}{12}$, they will be able to compare the fractions more easily.

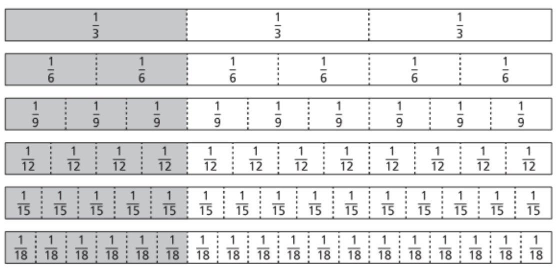
from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS

Benchmarks Students also reason using benchmarks such as $\frac{1}{2}$ and 1. For example, they see that $\frac{7}{8} < \frac{13}{12}$ because $\frac{7}{8}$ is less than 1 (and is therefore to the left of 1) but $\frac{13}{12}$ is greater than 1 (and is therefore to the right of 1).

Equivalent Fractions As students explore the number line, they observe that there are multiple ways to label the marks on a number line. This leads to a discussion that the different labels for the same point are equivalent fractions. For example, on these number lines, students can see that $\frac{1}{2}$ and $\frac{2}{4}$ are the same distance from 0, so $\frac{1}{2} = \frac{2}{4}$.

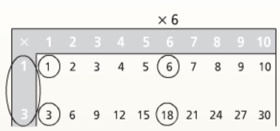


Fraction bar models and the multiplication table are also used to help students conceptualize equivalent fractions. In the fraction bar model below, students can see that the gray sections are the same length, so $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \frac{6}{18}$.



Students begin to see that, for example, the $\frac{1}{3}$ is partitioned into 2 smaller parts to get $\frac{2}{6}$, into 3 smaller parts to get $\frac{3}{9}$, into 4 smaller parts to get $\frac{4}{12}$, and so on. This partitioning idea helps them conceptualize why you can multiply and divide the numerator and denominator by forms of 1 ($\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, and so on) to find equivalent fractions. They connect their understanding of fraction bar models to the multiplication table to further solidify this understanding.

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100



$$\frac{1 \times 6}{3 \times 6} = \frac{6}{18}$$

$$\frac{6 \div 6}{18 \div 6} = \frac{1}{3}$$

Two rows from the multiplication table make series of equivalent fractions so that students see that these processes are general and can work for any numbers.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS

Equivalent Fractions Grade 4 students learn a fundamental property of equivalent fractions: multiplying the numerator and denominator of a fraction by the same non-zero whole number results in a fraction that represents the same number as the original fraction. This property forms the basis for much of their other work in Grade 4, including the comparison, addition, and subtraction of fractions and the introduction of finite decimals.

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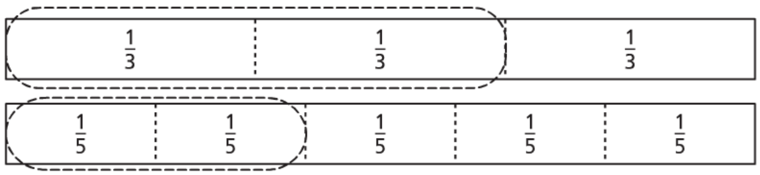
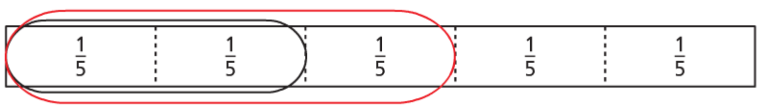
Partitioning Students can use area models and number line diagrams to reason about equivalence. They see that the numerical process of multiplying the numerator and denominator of a fraction by the same number, *n*, corresponds physically to partitioning each unit fraction piece into *n* smaller equal pieces. The whole is then partitioned into *n* times as many pieces, and there are *n* times as many smaller unit fraction pieces as in the original fraction.

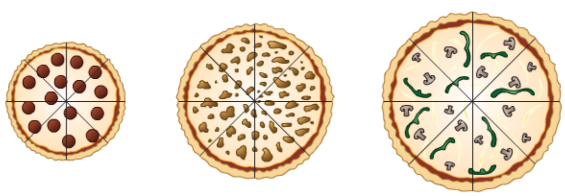
Comparing Fractions In Lesson 1, students revisit the concept of unit fractions. Students understand that the greater the number of parts (denominator), the smaller the fraction. They leverage this understanding to compare unit fractions and fractions with the same denominator or same numerator. For example, to compare $\frac{2}{3}$ and $\frac{2}{5}$, they reason that since fifths are smaller than thirds, 2 fifths is smaller than 2 thirds and $\frac{2}{5} < \frac{2}{3}$, or $\frac{2}{3} > \frac{2}{5}$. Fraction bar models are presented to help students visualize this concept.



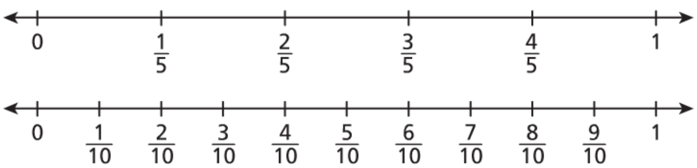
Fraction bars are also used to help students compare fractions with different numerators and the same denominator. This model shows that $\frac{2}{5} < \frac{3}{5}$.



Students also explore comparing fractions of different-sized wholes. Models help them visualize that, for example, $\frac{1}{8}$ of a bigger whole is greater than $\frac{1}{8}$ of a smaller whole.



Students use their understanding of the number line to identify that lesser fractions and mixed numbers are located to the left and greater fractions and mixed numbers are located to the right. So, on the number lines below, since $\frac{3}{5}$ is to the right of $\frac{5}{10}$, $\frac{3}{5} > \frac{5}{10}$.



Once students have developed a conceptual understanding of comparing fractions, they learn how to compare by finding equivalent fractions with a common denominator. Different strategies are introduced depending on the relationship between the denominators.

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Comparing Fractions Grade 4 students use their understanding of equivalent fractions to compare fractions with different numerators and different denominators. For example, to compare $\frac{5}{8}$ and $\frac{7}{12}$ they rewrite both fractions as $\frac{60}{96}$ $(= \frac{12 \times 5}{12 \times 8})$ and $\frac{56}{96}$ $(= \frac{7 \times 8}{12 \times 8})$ Because $\frac{60}{96}$ and $\frac{56}{96}$ have the same denominator, students can compare them using Grade 3 methods and see that $\frac{56}{96}$ is smaller, so $\frac{7}{12} < \frac{5}{8}$.

Case 1: One denominator is a factor of the other. **Possible Strategy:** Use the greater denominator as the common denominator.	**Example** Compare $\frac{3}{5}$ and $\frac{5}{10}$. Use 10 as the common denominator. $\frac{3 \times 2}{5 \times 2} = \frac{6}{10}$ $\frac{6}{10} > \frac{5}{10}$, so $\frac{3}{5} > \frac{5}{10}$.
Case 2: The only number that is a factor of both denominators is 1. **Possible Strategy:** Use the product of the denominators as the common denominator.	**Example** Compare $\frac{5}{8}$ and $\frac{4}{5}$. Use 5 × 8, or 40, as the common denominator. $\frac{5 \times 5}{8 \times 5} = \frac{25}{40}$ $\frac{4 \times 8}{5 \times 8} = \frac{32}{40}$ $\frac{25}{40} < \frac{32}{40}$, so $\frac{5}{8} < \frac{4}{5}$.
Case 3: There is a number besides 1 that is a factor of both denominators. **Possible Strategy:** Use a common denominator that is less than the product of the denominators.	**Example** Compare $\frac{5}{8}$ and $\frac{7}{12}$. 24 is a common multiple of 8 and 12. Use 24 as the common denominator. $\frac{5 \times 3}{8 \times 3} = \frac{15}{24}$ $\frac{7 \times 2}{12 \times 2} = \frac{14}{24}$ $\frac{15}{24} > \frac{14}{24}$, so $\frac{5}{8} > \frac{7}{12}$.

Decimal Concepts

Lessons 8 9 10 11 12

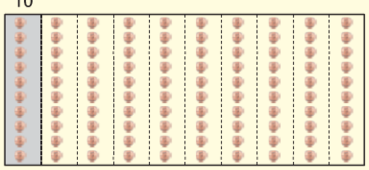
Models for Fractions and Decimals In Lesson 8, students use what they know about dimes, pennies, and dollars and fractional parts of a set to relate fractions and decimals. They learn that fraction and decimal numbers are different ways of writing the same value. The goal is for students to think of hundredths, whether written as a fraction or as a decimal, as pennies in a dollar and think of tenths as dimes in a dollar, when a dollar is used to represent 1 whole.

Students extend the models to represent halves, fourths, and numbers greater than 1. The following array models show the relationships between money, fractions, and decimals.

1 penny $= \frac{1}{100} = 0.01$

$\frac{10}{100}$ 10 of 100 equal parts

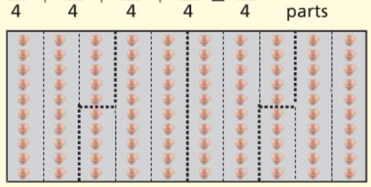
$\frac{1}{10}$ 1 of 10 equal parts



0.1

0.10

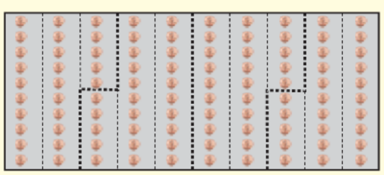
$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4}$ 4 of 4 equal parts = 1 whole

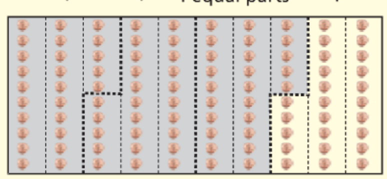


$0.25 + 0.25 + 0.25 + 0.25 = 1.00$

$\frac{25}{100} + \frac{25}{100} + \frac{25}{100} + \frac{25}{100} = \frac{100}{100}$

$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{7}{4}$ 7 of 4 equal parts $= 1\frac{3}{4}$



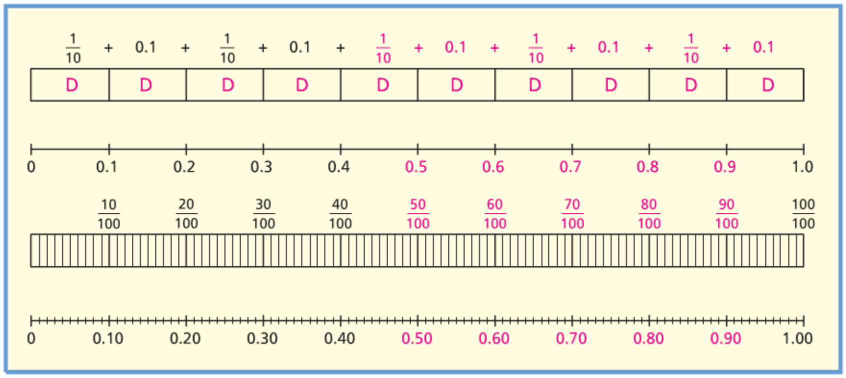


$\frac{4}{4} + \frac{3}{4}$

$0.25 + 0.25 + 0.25 + 0.25 + 0.25 + 0.25 + 0.25 = 1.75$

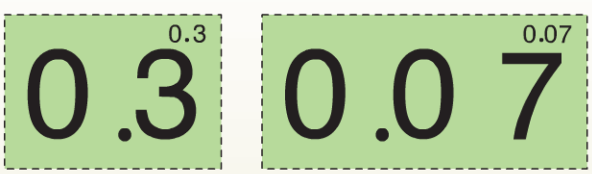
$\frac{25}{100} + \frac{25}{100} + \frac{25}{100} + \frac{25}{100} + \frac{25}{100} + \frac{25}{100} + \frac{25}{100} = \frac{175}{100} = \frac{100}{100} + \frac{75}{100} = 1 + \frac{75}{100} = 1\frac{75}{100}$

Students also explore decimal numbers and their fractional equivalents by using bars divided into tenths and hundredths. This bar model connects back to the fraction strips and number lines that students used to represent fractions. This model emphasizes thinking of fractions and decimals as parts of a whole. Tenths written as fractions and decimals are on a bar model divided into 10 equal parts. Hundredths written as fractions and decimals are on a bar divided into 100 equal parts.

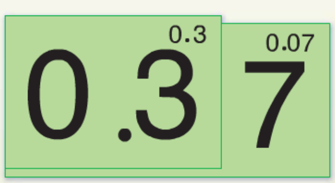


From the models above, students can visualize the relationship between tenths and hundredths and the equivalence between these numbers written as fractions and decimals. The D on the top bar model emphasizes that each section of the bar represents a dime, so the bar model also helps students connect the concept back to money.

Decimal Secret Code Cards Students explore decimal place value by assembling Secret Code Cards to form decimal numbers. The cards are used to show tenths, hundredths, and decimals both less than and greater than one. These cards help students remember the underlying concept of place value as they build decimal numbers. To make the decimal number 0.37, for example, students select the cards representing 3 tenths and 7 hundredths.



The 0.3 card is placed over the 0.07 card to create the number 0.37.



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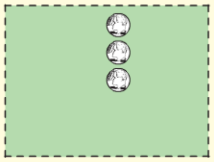
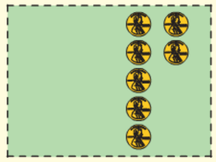
Decimals Fractions with denominator 10 and 100, called decimal fractions, arise naturally when students convert between dollars and cents, and have a more fundamental importance, developed in Grade 5, in the base 10 system. For example, because there are 10 dimes in a dollar, 3 dimes is $\frac{3}{10}$ of a dollar; and it is also $\frac{30}{100}$ of a dollar because it is 30 cents, and there are 100 cents in a dollar. Such reasoning provides a concrete context for the fraction equivalence

$$\frac{3}{10} = \frac{3 \times 10}{10 \times 10} = \frac{30}{100}$$

Secret Code Card Backs The backs of the cards show the corresponding number of dimes or pennies so students can make the connection to the models they used in Lesson 8.



3 dimes are on the back of the 0.3 card.



7 pennies are on the back of the 0.07 card.

Using the cards is beneficial for students because they are able to connect their understanding of whole number place value and the whole number Secret Code cards to the new concept of decimal place value.

Students also use the Secret Code Cards and the other models used in this unit, including their MathBoard, to help them compare decimal numbers. By looking at the back of the cards, students are able to visualize the comparison using the money representation. Lesson 12 shows students how to write zeros in decimal numbers so that the value of the number does not change, but so that it is easier to compare.

Problem:	Solution:
Which of these numbers is the greatest: 2.35, 2.3, or 2.4	2.35 With the places aligned 2.30 and the extra zeros added, 2.40 we can see which is greatest.

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Comparing Decimals Students compare decimals using the meaning of a decimal as a fraction, making sure to compare fractions with the same denominator. For example, to compare 0.2 and 0.09, students think of them as 0.20 and 0.09 and see that 0.20 > 0.09 because

$\frac{20}{100} > \frac{9}{100}$

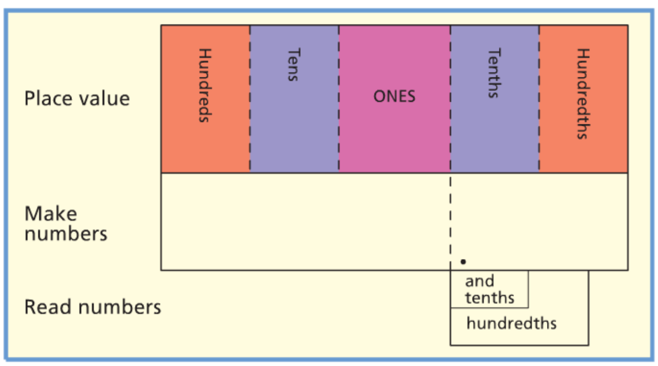
The argument using the meaning of a decimal as a fraction generalizes to work with decimals in Grade 5 that have more than two digits, whereas the argument using a visual fraction model, shown in the margin, does not. So it is useful for Grade 4 students to see such reasoning.

Decimal Place Value Lesson 11 presents a place value chart to help students understand how to show and read decimals. The *Math Expressions* place value chart is designed to emphasize that the "ones place" not the decimal point is the center of the chart. Students observe that tens and tenths and hundreds and hundredths are symmetric about the ones place.

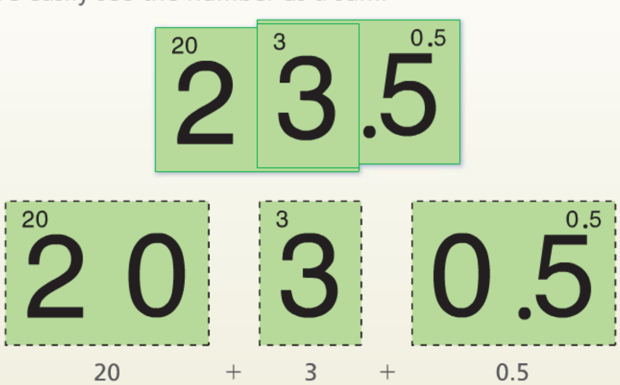
× 10 (Greater) ÷ 10 (Lesser)

Hundreds	Tens	ONES	Tenths	Hundredths
100.	10.	1.	0.1	0.01
$\frac{100}{1}$	$\frac{10}{1}$	$\frac{1}{1}$	$\frac{1}{10}$	$\frac{1}{100}$
$100.00	$10.00	$1.00	$0.10	$0.01

Students further model place value by placing the Decimal Secret Code Cards on the Place Value frame.



Using the frame is beneficial to students because it helps them make and read numbers. The cards also help students to write decimal numbers in expanded form. Notice that the value of the number on the card still appears when the numbers are assembled, so students can more easily see the number as a sum.



Problem Solving

Lessons 3 6 7

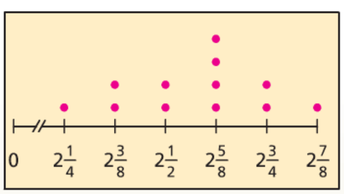
Problem Solving Plan In *Math Expressions* a research-based problem solving approach that focuses on problem types is used.

- Interpret the problem
- Represent the situation
- Solve the problem
- Check that the answer makes sense.

Real World Applications of Fractions and Decimals Throughout the unit, real world scenarios are used to develop meanings for fraction and decimal concepts. Students learn to solve problems involving finding factions of different-size wholes. They solve problems in which it is necessary to compare fractions or determine if fractions are equivalent. Students apply their understanding of decimal numbers to determine, for example, what part of a mile will be run in 8 days if 0.1 of a mile is run one day.

Line Plots In Unit 7, as well as in Unit 6, students apply their understanding of fractions to solve problems using line plots. They make line plots given fractional data and use the line plots to analyze the data and solve problems.

Width (in inches)	Number of Students
$2\frac{1}{4}$	1
$2\frac{3}{8}$	2
$2\frac{1}{2}$	2
$2\frac{5}{8}$	4
$2\frac{3}{4}$	2
$2\frac{7}{8}$	1



Hand Width (in inches)

Focus on Mathematical Practices

Lesson 13

The standards for Mathematical Practice are included in every lesson of this unit. However, there is an additional lesson that focuses on all eight Mathematical Practices. In this lesson, students use what they know about fractions and decimals to solve problems involving autumn leaves and walking trails.