Getting Ready to Teach Unit 4

Learning Path in the Common Core Standards

In this unit, students apply problem solving strategies and learn problem solving skills. Their work involves writing real world contexts, and writing situation and solution equations for one-step, two-step, and multistep problems representing addition, subtraction, multiplication, and division of whole numbers.

Visual models and real world situations are used throughout the unit to illustrate important number and operation concepts.

Help Students Avoid Common Errors

Math Expressions gives students opportunities to analyze and correct errors, explaining why the reasoning was flawed.

In this unit, we use Puzzled Penguin to show typical errors that students make. Students enjoy teaching Puzzled Penguin the correct way, why this way is correct, and why the error is wrong. The common errors are presented as letters from Puzzled Penguin to the students:

- **Lesson 2:** writing an incorrect situation equation
- **Lesson 5:** not choosing the correct operation
- **Lesson 8:** not following the Order of Operations when evaluating an equation with multiple operations
- **Lesson 11:** using the wrong rule to extend a geometric pattern

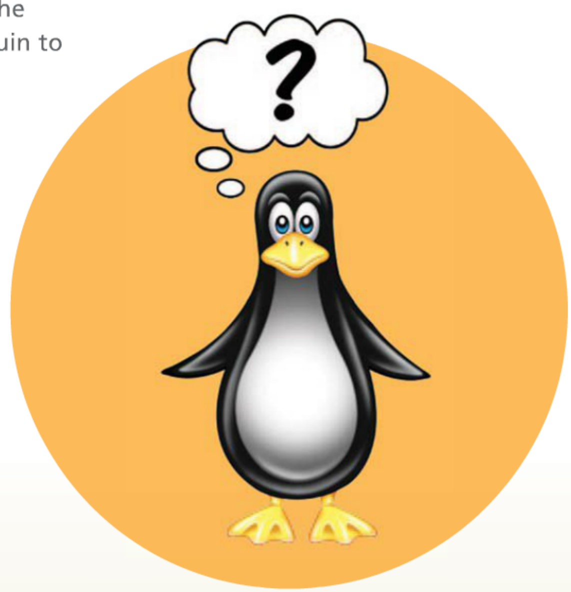
In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.

***Math Expressions* VOCABULARY**

As you teach this unit, emphasize understanding of these terms:

- situation equation
- solution equation
- comparison situation
- comparison bars

See the Teacher Glossary.



The Problem Solving Process

Using the Mathematical Practices Throughout the program, *Math Expressions* integrates a research-based algebraic problem solving approach that focuses on problem types. Problem solving is a complex process that involves all eight of the Common Core State Standards Mathematical Practices. It is also an individual process that can vary considerably across students. Students may conceptualize, represent, and explain a given problem in different ways.

Mathematical Practice	Student Actions
Understand the Problem Situation MP.1 Make sense of the problem. MP.2 Reason abstractly and quantitatively.	**Make Sense of the Language** Students use the problem language to conceptualize the real world situation.
Represent the Problem Situation MP.4 Model with mathematics. MP.7 Look for and make use of structure.	**Mathematize the Situation** Students focus on the mathematical aspects of the situation and make a math drawing and/or write a situation equation to represent the relationship of the numbers in the problem.
Solve the Problem MP.5 Use appropriate tools. MP.8 Use repeated reasoning.	**Find the Answer** Students use the math drawing and/or the situation or solution equation to find the unknown.
Check That the Answer Makes Sense MP.3 Critique the reasoning of others. MP.6 Attend to precision.	**Check the Answer in the Context of the Problem** Students write the answer to the problem, including a label. They explain and compare solutions with classmates.

Students are taught to make their own math drawings. Relating math drawings to equations helps them understand where the total and the product is for each operation, and helps them solve equations with difficult unknowns.

Math Talk Learning Community

Research In the NSF research project that led to the development of *Math Expressions*, much work was done with helping teachers and students build learning communities within their classrooms. An important aspect of doing this is Math Talk. The researchers found three levels of Math Talk that go beyond the usual classroom routine of students simply solving problems and giving answers and the teacher asking questions and offering explanations. It is expected that at Grade 4, students will engage in talk at all levels.

Math Talk Level 1 A student briefly explains his or her thinking to others. The teacher helps students listen to and help others, models fuller explaining and questioning by others, and briefly probes and extends students' ideas.

Example Word Problem

In a collection of 2,152 coins, 628 coins are pennies. How many coins are not pennies?

Who can tell us how many coins are not pennies?

Sarah: There are 1,524 coins that are not pennies.

How do you know?

Sarah: I know that 2,152 minus 628 is 1,524.

Who used a different method to answer the question?

Matthew: I did. I counted up from 628 to 2,152. 1,000 plus 500 more plus 24 more is 1,524.

Math Talk Level 2 A student gives a fuller explanation and answers questions from other students. The teacher helps students listen to and ask good questions, models full explaining and questioning (especially for new topics), and probes more deeply to help students compare and contrast methods.

Example Word Problem

Last weekend, Mr. Morgan rode his bike 3 miles. This weekend, he rode his bike 21 miles. How many times as many miles did Mr. Morgan ride his bike this weekend as last weekend?

How can we find the answer to this problem?

Stephi: We can draw a comparison bar diagram.

Steve: Is this a problem that has more than one answer?

Why do you ask that, Steve?

Steve: Because I know more than one way to draw a comparison bar diagram.

Stephi: The problem says "how many times as many miles" so I think that we have to use the diagram that shows multiplication.

Mary: I think so, too. If it was addition, it would say "how many more miles did he ride," not "how many times as many."

Stephi: Look at my diagram. It is the number of times that is unknown.

What operation do you think you need to use to solve the problem?

Mary: I guess division, because you have to divide the number of miles he road this weekend (21) by the number of miles he rode last weekend (3) to find out how many times more 21 is than 3.

Math Talk Level 3 The explaining student manages the questioning and justifying. Students assist each other in understanding and correcting errors and in explaining more fully. The teacher monitors and assists and extends only as needed.

Example Word Problem

Amy lives in the twentieth house on Elm Street. The first house on Elm Street is numbered 3. The second is 6. The third is 9. The fourth is 12. If this pattern continues, what is Amy's house number likely to be?

House	1st	2nd	3rd	4th	20th
Number	3	6	9	12	

Who will show us how to find the answer?

Shaina: I used the table to find the pattern. The house number goes up 3 each time, so I just have to start at the 4th house and add 3 until I get to the 20th house.

Jeff: I think you will find the answer that way, but it will take a long time. I wonder if there is a rule to go from the house to the house number?

Joel: I think there is! Look: 1 times 3 is 3, 2 times 3 is 6, 3 times 3 is 9, and 4 times 3 is 12. If you multiply the house by 3, you get the house number.

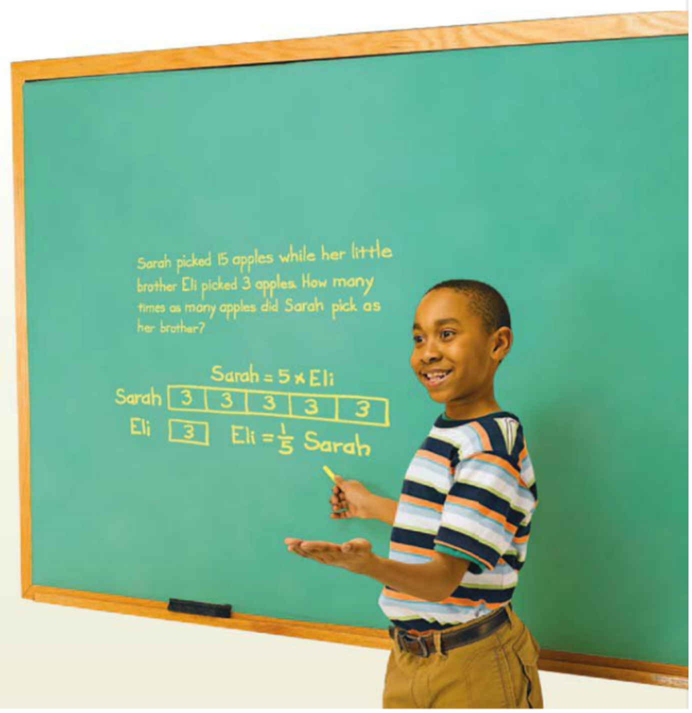
How can we use this rule to find the house number for the 20th house?

Jeff: I think I see it now. The rule is multiply the house number by 3. 20 times 3 is 50, so the 20th house will be 50.

Shaina: I think your rule works, but your answer did not match mine. I got 60. I am confused.

Joel: I think I see why. 20 times 3 is 60, not 50. See, count by 20 three times: 20, 40, 60. The rule was right, but you multiplied wrong.

Summary Math Talk is important not only for discussing solutions to word problems, but also for any kind of mathematical thinking students do, such as explaining why a number is prime or composite, how to use a drawing to subtract, or how to find the area of a composite figure.



Algebraic Relationships and Properties

Lesson 1

Order of Operations Unit 4 presents students with opportunities to solve a variety of problems. One strategy that students develop is writing an equation to solve not only one-step problems, but two-step, and multistep problems. For students to work flexibly with expressions and equations, they need to be able to use the correct Order of Operations. Lesson 1 presents the Order of Operations as a convention (or agreement) of how to perform operations. Students learn that they perform the operations inside the parentheses first, then multiplication and division as they appear from left to right, and then addition and subtraction as they appear from left to right. Becoming proficient in the use of the Order of Operations benefits students because it gives them the tools that they need to write and correctly simplify expressions and equations needed to solve more complicated problems.

Properties For students to become good problem solvers, they need to understand relationships among numbers. Lesson 1 presents students with properties of numbers, including the Identity Property, Distributive Property, Associative Property, and Commutative Property. For example, using the Identity Property and Distributive Properties, students are able to understand that:

$$n + 3n = 1 \cdot n + 3 \cdot n = (1 + 3) \cdot n = 4 \cdot n = 4n$$

Students apply these understandings as they represent and solve problems throughout the unit.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

An Algebraic Perspective Students thus begin developing an algebraic perspective many years before they will use formal algebraic symbols and methods. They read to understand the problem situation, represent the situation and its quantitative relationships with expressions and equations, and then manipulate that representation if necessary, using properties of operations and/or relationships between operations. Linking equations to concrete materials, drawings, and other representations of problem situations affords deep and flexible understandings of these building blocks of algebra.

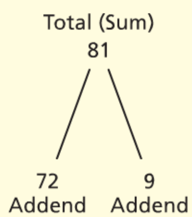
Addition and Subtraction

Lesson 2

The = and ≠ Signs If students are to be successful in using equations to represent problems, they need an understanding of the equals sign. Students sometimes see the equals sign as an indication of computation. For example, they will write adding 4 and 5 and then subtracting 3 as: $4 + 5 = 9 - 3 = 6$. This notation is incorrect since $4 + 5$ is not equal to $9 - 3$ and $4 + 5$ is not equal to 6. Students need to understand the equals sign as indicating a relationship, specifically, an equality relationship between the expressions on the left and right side of the equals sign. Giving students a variety of different types of equations, such as the ones below, will help students develop this understanding.

$5 = 3 + 2$ $\quad$ $3 + 2 = 5$ $\quad$ $5 = 5$ $\quad$ $3 + 2 = 2 + 3$ $\quad$ $7 - 2 = 1 + 1 + 3$

Inverse Operations Students build upon their understanding of addition and subtraction as inverse operations in Lesson 2. The break-apart drawing is used to represent this relationship. They learn that they can write 8 equations for each break-apart drawing, as shown in this example.



$81 = 72 + 9$	$72 + 9 = 81$
$81 = 9 + 72$	$9 + 72 = 81$
$72 = 81 - 9$	$81 - 9 = 72$
$9 = 81 - 72$	$81 - 72 = 9$

An understanding of inverse operations is beneficial for students as they learn to write and solve equations to model word problems.

Situation and Solution Equations for Addition and Subtraction In the beginning lessons in Unit 4, students write one or two equations to solve one-step word problems. Writing two equations involves writing a situation equation and a solution equation.

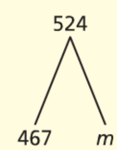
A *situation equation* shows the structure or relationship of the information in a problem. Students who write a situation equation to represent a problem use their understanding of inverse operations to then rewrite the equation as a solution equation.

A *solution equation* shows the operation that is used to solve a problem. If only one equation is written by students and used to solve a problem, the equation is a solution equation.

In Lesson 2, students model problems using break-apart drawings and write situation and solution equations to solve the problems.

Cassie had a collection of 467 marbles. Her grandfather gave here some more marbles. Now she has 524 marbles. How many marbles did Cassie get from her grandfather?

- The equation that shows the structure of the information in the problem is $467 + m = 524$.
- $467 + m = 524$ is the situation equation.
- The solution equation is $m = 524 - 467$.



Problem Types for Addition and Subtraction The following addition and subtraction problem types appear in *Math Expressions*.

Add to: Result Unknown

Jamal put a cantaloupe with a mass of 450 grams in a bag. He adds another cantaloupe that has a mass of 485 grams. How many grams of cantaloupe are in the bag?

Add to: Change Unknown

The workers at a factory made 3,250 pink balloons in the morning. There were 5,975 pink balloons at the factory at the end of the day. How many pink balloons did the factory workers make in the afternoon?

Add to: Start Unknown

There were some people at the football stadium early last Sunday, and then 5,427 more people arrived. Then there were 79,852 people at the stadium. How many people arrived early?

Take from: Result Unknown

Terrence is planning a 760-mile trip. He travels 323 miles the first two days. How many miles does Terrence have left to travel on this trip?

Take from: Change Unknown

At the end of a baseball game, there were 35,602 people in the stadium. There were 37,614 people in the stadium at the beginning of the game. How many people left before the game ended?

Take from: Start Unknown

Susanna took $3,050 out of her bank account. Now she has $11,605 left in the account. How much money was in Susanna's account to start?

Put Together/Take Apart: Total Unknown

In the month of May, Movieland rented 563 action movies and 452 comedy movies. How many action and comedy movies in all did Movieland rent in May?

Put Together/Take Apart: Addend Unknown

In a collection of 2,152 coins, 628 coins are pennies. How many coins are not pennies?

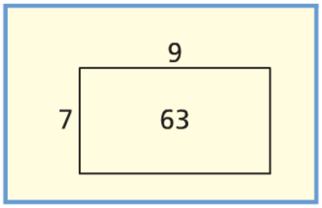
***from* THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING**

Problem Types In Grades 3, 4, and 5, students extend their understandings of addition and subtraction problem types in Table 1 to situations that involve fractions and decimals. Importantly, the situational meanings for addition and subtraction remain the same for fractions and decimals as for whole numbers.

Multiplication and Division

Lesson **3**

Relating Division and Multiplication Lesson 3 presents students with the opportunity to expand their knowledge of multiplication and division by exploring the inverse relationship between the two operations. A rectangle model is used to help students see the relationship between factors, product, quotient, dividend, and divisor. In a rectangle model, the product (or dividend) is placed inside the rectangle. The factors (quotient and divisor) are above and to the left of the rectangle, representing its length and width.



Students learn that from the rectangle model above, they can write the following related multiplication and division number sentences.

$63 = 7 \times 9$	$7 \times 9 = 63$
$63 = 9 \times 7$	$9 \times 7 = 63$
$7 = 63 \div 9$	$63 \div 9 = 7$
$9 = 63 \div 7$	$63 \div 7 = 9$

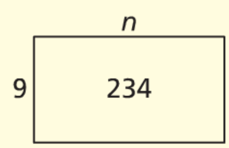
A rectangle model can help students see the information that is unknown and recognize the operation that is used to find that information. For example, when a completed rectangle shows a product in the middle and a factor as the length or width, students recognize division as the operation that is used to find an unknown factor.

Students build upon their understanding of inverse operations by using a rectangle model as a springboard for writing the situation and solution equations that are used to solve a multiplication or division problem involving whole numbers.

Situation and Solution Equations for Multiplication and Division

Multiplication and division contexts are presented in Lesson 3. To help recognize and generate the situation and solution equations that represent the various contexts, students model the contexts using rectangle models. In the example below, students use a rectangle model to represent a problem about the number of trees in each row.

Brenda planted 234 trees on her farm. The farm has 9 rows of trees. How many trees are in each row?



Notice that one of the dimensions of the rectangle is unknown, so students use a variable, for example n, to represent the unknown dimension in the rectangle. From the rectangle model, students see that they can write the situation equation:

$$n \cdot 9 = 234$$

and the solution equation:

$$234 \div 9 = n$$

Problem Types for Multiplication and Division Problems

The following multiplication and division problem types appear in *Math Expressions.*

Equal Groups: Unknown Product

> Al's photo album has 26 pages. Each page has 4 photos. How many photos are in Al's album?

Equal Groups: Unknown Product (Measurement Example)

> Evan is starting a cycling program. He will ride 315 miles each month for the next 6 months. How many miles does he plan to ride in all?

Equal groups: Group Size Unknown

> The Tropical Tour Company has 2,380 brochures to distribute equally among its 7 resort hotels. How many brochures will each hotel receive?

Equal groups: Group Size Unknown (Measurement Example)

> Tanisha has 35 milliliters of apple juice. She divides the juice evenly into 5 different glasses. How much juice does she pour into each glass?

Equal Groups: Number of Groups Unknown

> Suki has 152 stickers to place in a sticker album. How many pages will Suki fill with stickers if each page in the album holds 8 stickers?

Equal Groups: Number of Groups Unknown (Measurement Example)

> Thomas has one 9-foot pine board. He needs to make 4-foot shelves for his books. How many shelves can he cut?

Problem Types for Multiplication and Division Problems (continued)

Arrays: Unknown Product

> Maria's father planted 12 rows of tomatoes in his garden. Each row had 6 plants. How many tomato plants were in Maria's father's garden?

Area: Unknown Product (Area Unknown)

> Casey draws a rectangular array that is 1,167 units long and 7 units wide. What is the area of Casey's array?

Arrays: Group Size Unknown

> Brenda planted 234 trees on her farm. The farm has 9 rows of trees. How many trees are in each row?

Area: Group Size Unknown (Bigger Side Unknown)

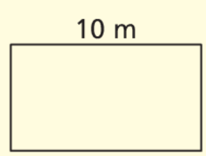
> A landscape architect designs a rectangular garden that is 1,232 square feet. It is 8 feet wide. How long is the garden?

Arrays: Number of Groups Unknown

> Terrence has 24 model cars arranged in equal rows of 6 model cars. Natalie has 18 model cars arranged in equal rows of 3 model cars. How many rows of model cars in all do they have?

Area: Number of Groups Unknown (Smaller Side Unknown)

> The area of the rectangle is 60 square meters. One side of the rectangle has a length of 10 meters. What is the unknown side length?

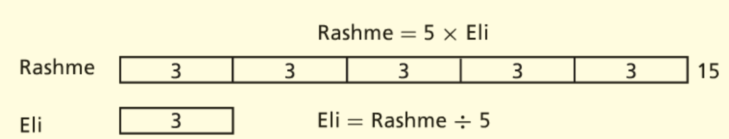


Multiplicative and Additive Comparisons

Lessons **4** **5** **6**

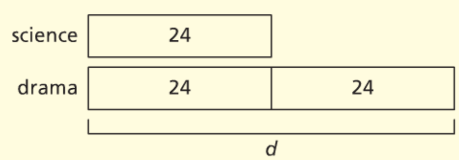
Multiplicative Comparison Problems Relating math drawings to equations helps students understand where the total and the product is for each operation. This helps students solve equations with unexpected unknowns. This is especially true for multiplication comparison problems, where students easily reverse the relationships. Making a drawing of comparison bars helps students show which quantity is larger and then decide on a correct solution equation or computation.

In the example below, the comparison bar drawing shows that Rashme picked 5 times as many apples as Eli. This relationship can be written in two ways; in terms of Rashme: Rashme = 5 · Eli, or in terms of Eli: Eli = Rashme ÷ 5.

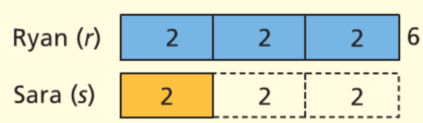


Students are encouraged to draw comparison bar models for the problems in these lessons.

There are 24 students in the science club. There are 2 times as many students in the drama club. How many students are in the drama club?



To prepare for a family gathering, Sara and Ryan made soup. Sara made 2 quarts. Ryan made 6 quarts.



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Comparison Problems In an additive comparison problem, the underlying question is what amount would be added to one quantity in order to result in the other. In a multiplicative comparison, the underlying question is what factor would multiply one quantity in order to result in the other.

Multiplicative Comparison Problem Types In Lessons 4, 5, and 6, students solve a variety of comparison problems involving multiplication of whole numbers.

Compare: Unknown Product

> There are 24 students in the science club. There are 2 times as many students in the drama club. How many students are in the drama club?

Compare: Unknown Product (Measurement Example)

> Elena has a cat with a mass of 4 kilograms. Ginger's cat has a mass that is 2 times as much as Elena's cat. What is the mass of Ginger's cat in grams?

Compare: Group Size Unknown

> There are 180 pennies in Miguel's coin collection and that is 5 times as many as the number of quarters in his coin collection. How many quarters does Miguel have?

Compare: Group Size Unknown (Measurement Example)

> Audrey has 1,263 centimeters of fabric, and that is 3 times as much fabric as she needs to make some curtains. How many centimeters of fabric does Audrey need to make the curtains?

Compare: Number of Groups Unknown

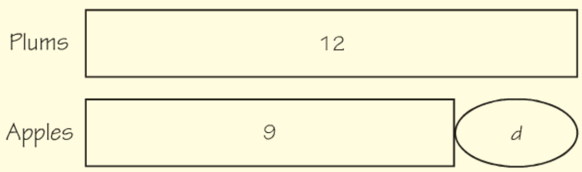
> Fred has 72 football cards and Scott has 6 football cards. How many times as many football cards does Fred have as Scott has?

Compare: Number of Groups Unknown (Measurement Example)

> Last weekend, Mr. Morgan rode his bike 3 miles. This weekend, he rode his bike 21 miles. How many times as many miles did Mr. Morgan ride his bike this weekend as last weekend?

Additive Comparison Problems In Lesson 5, students explore additive comparison situations. They also compare additive and multiplicative comparison problems, learning how to distinguish between the two. Students may conceptualize, represent, and explain a given problem in different correct ways. This is especially true for additive comparison problems, where students easily reverse the relationships. Making a drawing helps students show which quantity is larger and then decide on a correct solution, equation, or computation. Students often prefer to show an addition equation Smaller + Difference = Larger rather than the subtraction equation shown in some books as the only representation for comparison situations. Students are encouraged to use comparison bar drawings to help them model additive comparison problems. The model below represents the relationship expressed in this situation:

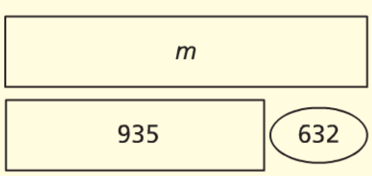
Mai has 9 apples and 12 plums.



Students can use the model to answer a variety of questions including: How many fewer apples than plums does Mai have? and How many more plums than apples does Mai have? From this model, students can more easily determine that they need to subtract to find the unknown.

The unknown amount can be in any of the three parts in the comparison diagram. For example, in the following problem, the total is unknown, so the longest rectangle represents the unknown. For the problem below, the diagram helps students determine that they need to add to find the answer.

The school fair fundraiser made $632 more from baked goods than from games. The school fair made $935 from games. How much money did the school fair make from baked goods?



Additive Comparison Problem Types In Lessons 5 and 6, students solve a variety of comparison problems involving addition of whole numbers.

Compare: Difference Unknown (How many more?)

> Dan wants to plant 30 trees. He has dug 21 holes. How many more holes does Dan need to dig?

Compare: Difference Unknown (How many fewer?)

> A nursery has 70 rose bushes and 50 tea-tree bushes. How many fewer tea-tree bushes than rose bushes are at the nursery?

Compare: Bigger Unknown (Version with more)

> The school fair fundraiser made $632 more from baked goods than from games. The school fair made $935 from games. How much money did the school fair make from baked goods?

Compare: Bigger Unknown (Version with fewer)

> Samantha has 145 fewer songs on her portable media player than Luke has on his portable media player. If Samantha has 583 songs, how many songs does Luke have?

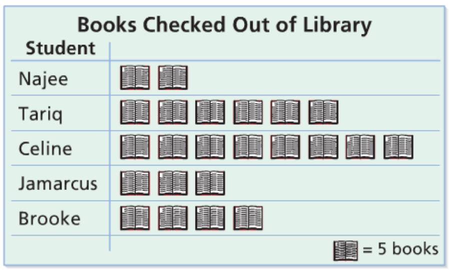
Compare: Smaller Unknown (Version with more)

> The soccer team drilled for 150 minutes last week. The team drilled for 30 minutes more than it scrimmaged. For how long did the team scrimmage?

Compare: Smaller Unknown (Version with fewer)

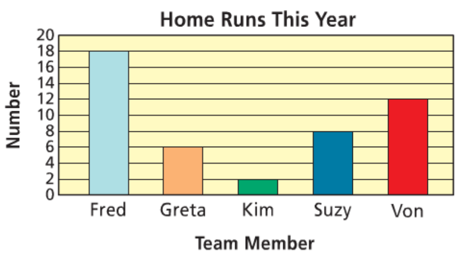
> Kyle and Mackenzie are playing a computer game. Kyle scored 7,628 points. Mackenzie scored 2,085 fewer points than Kyle. How many points did Mackenzie score?

Comparison Problems and Data Displays In Lesson 6, students solve comparison problems by interpreting data on data displays. Bar graphs and pictographs are designed to help students visualize comparison relationships. For example, the height of two bars can cue students as to whether one quantity is twice another. If one bar is twice as tall as another, the quantities share the same relationship. Using data displays, students can solve both additive and multiplicative comparison problems.



Using this pictograph, students can solve problems such as the following:

- *Which student checked out 30 fewer books than Celine?*
- *Celine checked out twice as many books as which student?*



Using this bar graph, students can solve problems such as the following:

- *How many more home runs did Von hit than Greta?*
- *This year Fred hit twice as many home runs as he hit last year. How many home runs did Fred hit last year?*

Problems with More Than One Step

Lessons 7 8 9

Different Approaches for Solving Problems There are multiple entry points to solving two-step or multistep problems. There are no algorithmic one-way approaches to all problems. We do not want to discourage individual ways to approach a given problem, because that is what may lead in the future to new solutions to new problems not yet even imagined. For a given problem, some students may use a forward approach, almost automatically finding an answer for a first part of the problem and then deciding if that is a step toward the needed overall solution (is it the answer to a first-step question?). Others may work backwards from the given problem question. Students may write two or more equations, or make one or more math drawings.

All of the problems in Lessons 1–6 involved one step. The problems in Lessons 7–9 involve more than one step. Problems that involve more than one step typically involve more than one operation. Sometimes when more than one operation is present, the order in which the operations are performed is important. As students work through the problems in these lessons, they will apply the understanding of the Order of Operations that they developed earlier in the unit.

Two-Step Problems In Lesson 7, students solve two-step problems. They practice identifying the "helping question" or the question that needs to be answered before the final solution can be found. Different solution equations are emphasized. For example, students might use two equations to represent the two operations involved in the solution, or they may write one equation. Students' prior understanding of parentheses is leveraged as they evaluate complex equations by completing the operations inside the parentheses first.

from THE OPERATIONS AND ALGEBRAIC THINKING PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS

Two-Step and Multistep Problems
Students use the new kinds of numbers, fractions, and decimals, in geometric measurement and data problems and extend to some two-step and multistep problems involving all four operations. In order to keep the difficulty level from becoming extreme, there should be a tradeoff between the algebraic or situational complexity of any given problem and its computational difficulty taking into account the kinds of numbers involved.

Two-Step Problems The two-step problems students solve in this lesson include whole numbers. Notice that two methods of finding the answer are presented.

At Parkes Elementary School, there are 6 fourth-grade classes with 17 students in each class. On Friday, 23 fourth-graders brought lunch from home and the rest of the students bought lunch from the cafeteria. How many fourth-graders bought lunch from the cafeteria on Friday?

Tommy's Method

Write an equation for each step.

Find the total number of students who are in fourth grade.

$6 \times 17 = \underline{102}$

Subtract the number of students who brought lunch from home.

$102 - 23 = \underline{79}$

Lucy's Method

Write an equation for the whole problem.

Let n = the number of students who bought lunch.

Students in each fourth grade class. → 17

Students who brought lunch from home. → 23

$6 \times \underline{17} - \underline{23} = n$

$\underline{79} = n$

Multistep Problems In Lesson 8, multistep problems are presented. In multistep problems, more than two steps must be completed to generate the solutions. In these types of problems, more than one helping question needs to be answered before the final solution can be obtained. Again, students learn that they can write several equations to represent the problem, or one equation with several operations. A broad understanding of the Order of Operations, including the use of parentheses, is essential for students to be successful at solving multistep problems.

Multistep Problems The multistep problems solved in this lesson include whole numbers. Notice that two methods of finding the answer are presented.

Mr. Stills makes bags of school supplies for the 9 students in his class. He has 108 pencils and 72 erasers. Each bag has the same number of pencils and the same number of erasers. How many more pencils than erasers are in each bag of school supplies?

Nicole's Method

Write an equation for each step.

Divide to find the number of pencils that Mr. Stills puts in each bag of school supplies. $108 \div 9 = \underline{12}$

Divide to find the number of erasers that Mr. Stills puts in each bag of school supplies. $72 \div 9 = \underline{8}$

Subtract the number of erasers in each bag from the number of pencils in each bag. $12 - 8 = \underline{4}$

There are __4__ more pencils than erasers in each bag of school supplies.

David's Method

Write an equation for the whole problem.

Let p = how many more pencils than erasers are in each bag of school supplies

The number of pencils in each bag of school supplies. | The number of erasers in each bag of school supplies.

$$\underline{108} \div 9 - \underline{72} \div 9 = p$$

$$12 - 8 = p$$

$$\underline{4} = p$$

There are __4__ more pencils than erasers in each bag of school supplies.

Factors, Multiples, Prime and Composite Numbers, and Patterns

Lessons 10 11

Factors and Factor Pairs To aid students in solving problems involving multiplication and division, it is beneficial for them to develop a broad understanding of factors and factor pairs. Understanding that a pair of numbers is a factor pair of another number can help students solve problems. For example, if students know that 4 and 25 are a factor pair of 100, they can more easily solve problems in which the solution is the quotient of 100 and 25 or 100 and 4. Developing fluency with identifying all of the factor pairs for numbers also benefits students because it enables students to more easily solve problems in which they need to partition a quantity into equal groups, for example, arranging a certain number of chairs in equal rows.

Prime and Composite Numbers Constructing arrays helps students conceptualize the difference between prime and composite numbers. Rectangular arrays help students visualize that a prime number has only two factors, one and itself, and so can be only represented by two arrays. For example, 13 can be represented only by an array that is 1 by 13 and one that is 13 by 1. A composite number, such as 24, on the other hand, can be represented by more than 2 arrays.

Students make arrays with 24 counters, like the one above, to show that 24 is a composite number. They can also make arrays that are 3 by 8, 4 by 6, 6 by 4, 8 by 3, 12 by 2, 24 by 1 and 1 by 24.

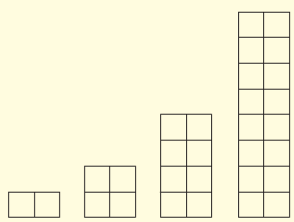
Multiples In Lesson 10, the relationship between factors and multiples is explored. Students learn that a multiple of a number is a product of that number and a counting number. They apply the understanding of count-bys that they developed in earlier grades to name multiples. For example, they count by 7 to name the multiples of 7: 7, 14, 21, 28, 35, and so on. Students will expand and apply their understanding of multiples as they explore fractions.

Patterns In Lesson 11, students explore a variety of numerical and geometric patterns. They extend patterns, as well as write a general rule that can be used to describe the pattern. Students use these skills to solve problems involving patterns, like the ones below:

Amy lives in the twentieth house on Elm Street. The first house on Elm Street is numbered 3. The second is 6. The third is 9. The fourth is 12. If this pattern continues, what is Amy's house number likely to be?

House	1st	2nd	3rd	4th	20th
Number	3	6	9	12	

Describe the number of squares in the next term in the pattern.



Focus on Mathematical Practices

Lesson 12

The Standards for Mathematical Practice are included in every lesson of this unit. However, there is an additional lesson that focuses on all eight Mathematical Practices. In this lesson, students write equations to solve problems about pottery.