Getting Ready to Teach Unit 3

Learning Path in the Common Core Standards

In Unit 2, students developed an understanding of multiplication of multidigit numbers. They apply this understanding in this unit as they explore the concept of division of numbers through thousands, with and without remainders. Students relate the multiplication models and methods to division as they learn various ways to divide.

The activities in this unit help students gain a practical understanding of methods of division and the role of estimation in determining quotients. Students also use this understanding of division to solve a variety of word problems, including problems in which they have to interpret the remainder.

***Math Expressions* VOCABULARY**

As you teach the unit, emphasize understanding of these terms.

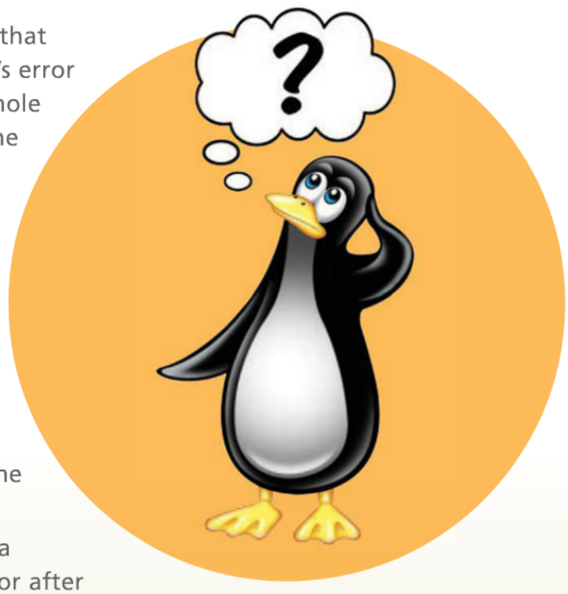
- Place Value Sections Method
- Expanded Notation Method
- Digit-by-Digit Method

See the Teacher Glossary

Help Students Avoid Common Errors

Math Expressions gives students opportunities to analyze and correct errors, explaining why the reasoning was flawed.

In this unit, we use Puzzled Penguin to show typical errors that students make. Students enjoy explaining Puzzled Penguin's error and teaching Puzzled Penguin the correct way to divide whole numbers. The following common errors are presented to the students as letters from Puzzled Penguin and as problems in the Teacher Edition that were solved incorrectly by Puzzled Penguin.



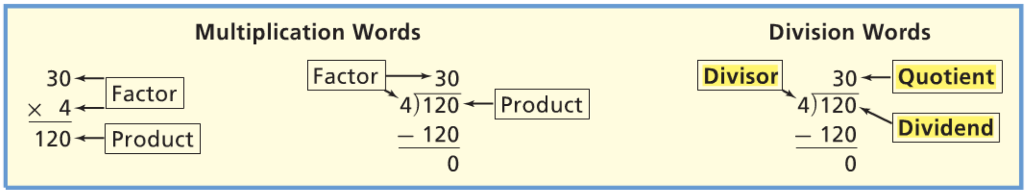
- **Lesson 2:** Incorrectly determining the place value of the digits in the quotient when using the Expanded Notation Method
- **Lesson 4:** Misplacing the first digit of a quotient when using the Digit-by-Digit Method of division
- **Lesson 5:** Incorrectly determining the place values of the digits in the quotient
- **Lesson 7:** When choosing digits in the quotient, using a multiplier that leaves a quantity greater than the divisor after the subtraction step

In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As a part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.

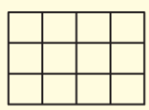
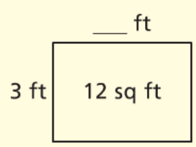
Understanding Division With and Without Remainders

Lesson 1

Multiplication and Division Vocabulary In Unit 2, students learned about multiplication. In this unit, students apply that understanding to the concept of division. They learn that the multiplication vocabulary they know, for example, product and factor, relate directly to division terms:



Using Arrays and Area Models for Division In Unit 2, arrays and area models were used to represent multiplication. Students conceptualized the area model as a pushed-together array. The models that students use in this unit to represent division are parallel to the models they used to represent multiplication.

array	rows and columns	area model
		
12 bottles on a table	12 tiles on a wall	12 square feet of carpet
12 ÷ 3 = __4__	12 ÷ 3 = __4__	12 ÷ 3 = __4__
3 × __4__ = 12	3 × __4__ = 12	3 × __4__ = 12

All of the models above can be used to represent 12 ÷ 3 = 4. Notice how context is used to clarify the meaning of the model for students. In the area model, the area is given and written inside the rectangle, one measurement is given, and the other is missing. Working with these familiar multiplication models within the context of familiar basic facts facilitates students' ability to apply their previous understanding of multiplication to the concept of division.

Remainders In this unit, students learn that if one number does not divide another number evenly, there is a remainder. They apply their already existing knowledge of basic facts to help them learn this new idea.

In the example below, students utilize their knowledge of basic multiplication facts to reason that since 4 • 3 = 12, and 12 + 2 = 14, 14 ÷ 3 = 4 remainder 2.

$$\begin{array}{r} 4 \text{ R2} \\ 3\overline{)14} \\ -12 \\ \hline 2 \end{array} \quad \leftarrow \text{2 are left over.}$$

Furthermore, students conceptualize why the remainder must be less than the divisor and that they can multiply the quotient by the divisor and add the remainder to check their answer.

Division with Zeros In Unit 2, students learned how to use place value and patterns to multiply one-digit numbers by multiples of 10, 100, and 1,000. In this unit, students relate this understanding of patterns in multiplication to conceptualize how to divide multiples of 10, 100, and 1,000 by 1-digit numbers. Pattern charts like the ones below help students solidify their reasoning.

4 × 1 = 4 4 × 10 = 40 4 × 100 = 400 4 × 1,000 = 4,000	4 ÷ 4 = 1 40 ÷ 4 = 10 400 ÷ 4 = 100 4,000 ÷ 4 = 1,000

7 × 5 = 35 7 × 50 = 350 7 × 500 = 3,500 7 × 5,000 = 35,000	35 ÷ 7 = 5 350 ÷ 7 = 50 3,500 ÷ 7 = 500 35,000 ÷ 7 = 5,000

***from* THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN**

Remainders Multi-digit division requires working with remainders. In preparation for working with remainders, students can compute sums of a product and a number, such as 4 • 8 + 3.

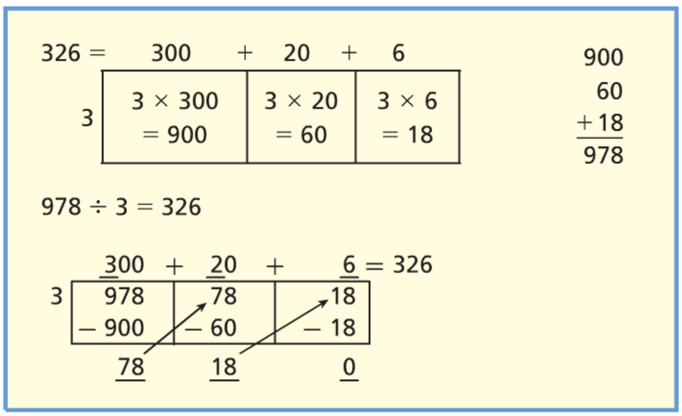
***from* THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN**

Division Patterns One component is quotients of multiples of 10, 100, or 1000 and one-digit numbers. For example, 42 ÷ 6 is related to 420 ÷ 6 and 4200 ÷ 6. Students can draw on their work with multiplication and they can also reason that 4200 ÷ 6 means partitioning 42 hundreds into 6 equal groups, so there are 7 hundreds in each group.

Three-Digit Quotients

Lesson **2**

Relate Multiplication Methods to Division Methods Just as Unit 2 presented students with a variety of numeric multiplication methods, this unit presents a variety of numeric division methods. The methods can be represented using the same area model used when multiplying. In all of the methods, the digit in each place of the dividend is divided by the divisor. However, the manner in which the division steps are recorded varies. The parallel structure of the multiplication and division methods is beneficial for students because it allows them to continue to develop conceptual understanding of operations and how they are related. The examples below show how the Place Value Sections Method is applied to a multiplication: $3 \cdot 326 = 978$ and the related division: $978 \div 3 = 326$.
$3 \times 326 = 978$

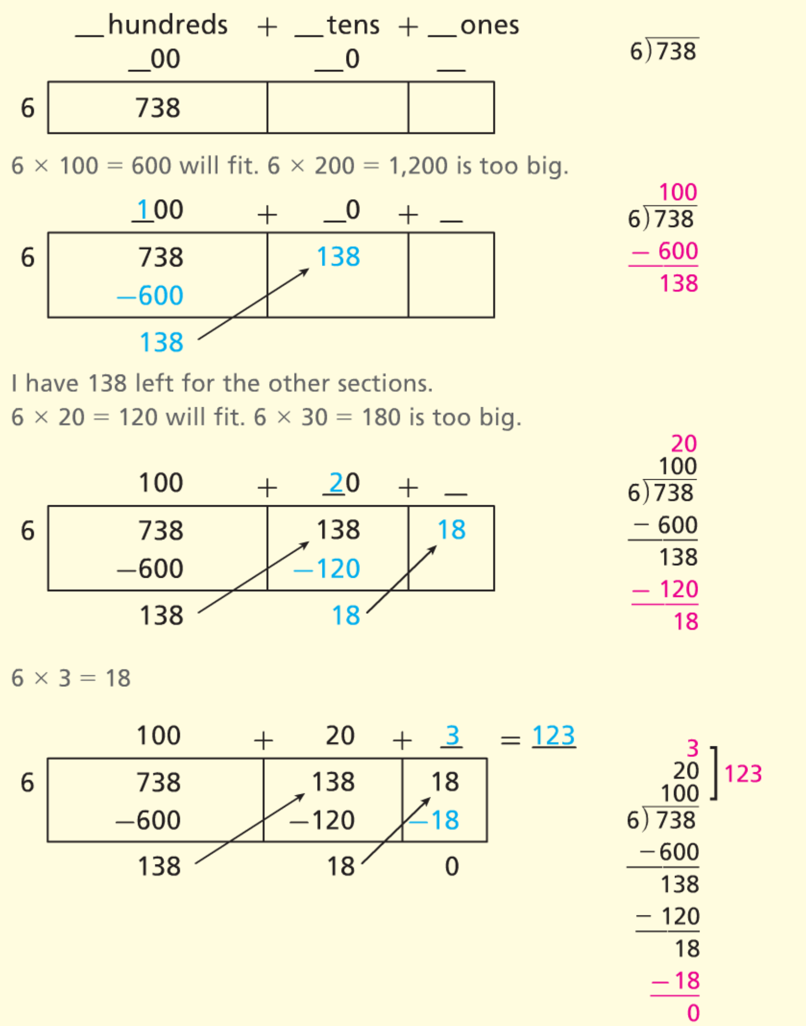


from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

Relate Multiplication and Division Methods General methods for computing quotients of multi-digit numbers and one-digit numbers rely on the same understandings as for multiplication, but cast in terms of division.

Place Value Sections and Expanded Notation Methods As with multiplication, students use both of these models to continue to broaden their experience of modeling division. The following examples show how students divide 738 by 6 using these methods. Students apply the Distributive Property as they think of the dividend as the sum of the values of its digits, for example, 738 = 700 + 30 + 8.

Place Value Sections Method | **Expanded Notation Method**



Notice that the multiplication and subtraction, necessary to complete the problem, are the same in each method, but recorded in different ways. In Lesson 2, students focus on *measurement* division. In this type of division, a total is divided into groups of a certain size and the number of groups needs to be determined.

from **THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN**

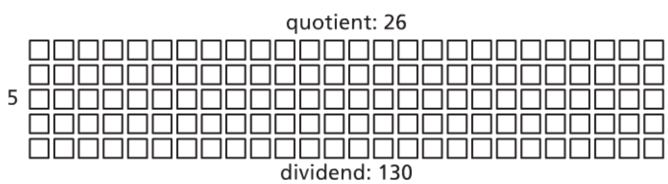
Division Models and Methods
Another component of understanding general methods for multidigit division computation is the idea of decomposing the dividend into like base-ten units and finding the quotient unit by unit, starting with the largest unit and continuing on to smaller units. As with multiplication, this relies on the distributive property. This can be viewed as finding the side length of a rectangle (the divisor is the length of the other side) or as allocating objects (the divisor is the number of groups).

Two- and Four-Digit Quotients

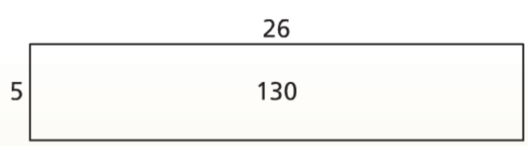
Lesson **3**

Using the Area Model for Division In Lesson 3, students focus on *partitive* division, in which a total is divided into a certain number of groups and the number in each group needs to be determined. Students consider problems such as *A grocer has 130 cans to put on 5 shelves. How many cans will fit on each shelf?* The 5-by-1 array models placing one can on each shelf. In this problem, the number of shelves is the divisor.

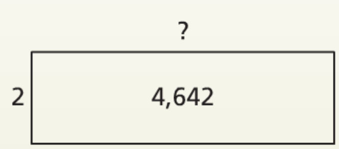
They apply their understanding of multiplication modeling to represent the division with an array.



Students realize that as dividends increase, sketching the large number of squares necessary to represent the greater dividend becomes less efficient. So, they apply their understanding that a rectangle model can be used to model the problem the same way they used a rectangle model in multiplication. The placement of the values is purposeful to correlate to the long division bracket notation.



As with the other numerical methods, students generalize that the model can be used for greater numbers. For example, this model is used to represent 4,642 ÷ 2.



Equal Groups As students experience with division deepens, they begin to think of division as equally distributing a given number of hundreds, tens, and ones into a given number of groups. This idea of equal grouping helps them to record the steps of their division. The example below shows how students use equal groups to divide 516 by 4. Notice that equal groups of hundreds are made, then subtracted from the total. Then equal groups of tens are made with the amount that is left and subtracted, and so on. It is in the comprehension of equal groups in division that students gain the ability to generalize division methods and can apply them to any multidigit division problem.

$516 \div 4 = ?$

4 groups

4)516

Divide 5 hundreds, 1 ten, 6 ones equally among 4 groups.

Complete the steps.

Step 1

4 groups

1 hundred
1 hundred
1 hundred
1 hundred

5 hundreds ÷ 4

each group gets 1 hundred; 1 hundred is left.

```
   1
4)516
 -4
  1
```

Regroup 1 hundred.

Now, I have 10 tens + 1 ten = 11 tens.

```
   1
4)516
 - 4
   11
```

Step 2

4 groups

1 hundred + 2 tens
1 hundred + 2 tens
1 hundred + 2 tens
1 hundred + 2 tens

11 tens ÷ 4

each group gets 2 tens; 3 tens is left.

```
   12
4)516
 -4
  11
 - 8
   3
```

Regroup 3 tens.

Now, I have 30 ones + 6 ones = 36 ones.

```
   12
4)516
 -4
  11
 - 8
   36
```

Step 3

4 groups

1 hundred + 2 tens + 9
1 hundred + 2 tens + 9
1 hundred + 2 tens + 9
1 hundred + 2 tens + 9

36 ones ÷ 4

each group gets 9 ones; There are 0 ones left.

```
   129
4)516
 -4
  11
 - 8
   36
 - 36
    0
```

There are 129 apples in each crate.

from **THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN**

Equal Groups In multi-digit division, students will need to find the greatest multiple less than a given number. For example, when dividing by 6, the greatest multiple of less than 50 is $6 \cdot 8 = 48$. Students can think of these "greatest multiples" in terms of putting objects into groups. For example, when 50 objects are shared among 6 groups, the largest whole number of objects that can be put in each group is 8, and 2 objects are left over. (Or when 50 objects are allocated into groups of 6, the largest whole number of groups that can be made is 8, and 2 objects are left over.) The equation $6 \cdot 8 + 2 = 50$ (or $8 \cdot 6 + 2 = 50$) corresponds with this situation.

Developing Fluency

Lessons **4** **5** **6** **7**

Digit-by-Digit Method In Lesson 4, students learn how to record division using the Digit-by-Digit method. This method is based on their understanding of the equal groups concept of division. Similar to the Shortcut Method in multiplication, in this method, students use a shortcut way of recording the sub-quotients. They imagine the zeros, rather than writing them. The following shows how to find 948 ÷ 4 using both methods.

Expanded Notation Method	Digit-by-Digit Method
7	237
30] 237	4)948
200	− 8
4)948	14
− 800	− 12
148	28
− 120	− 28
28	0
− 28	
0	

Notice that the sub-quotient 200 is written as a 2 in the hundreds place, the sub-quotient 30 is written as a 3 in the tens place, and the sub-quotient 7 is written as a 7 in the ones place.

Comparing Methods The complexity of division makes it necessary for students to have ample opportunity to use a variety of division methods so that they can gain a deeper understanding of the concept of division. The work that students do in Grade 4 will provide them with the foundation that they needs as they move on to Grade 5 where they will be expected to develop fluency with multidigit division, including division with two-digit divisors.

Zeros in Quotients As students divide a variety of numbers, they are presented with problems in which there are zeros in the dividend or in the quotient. The work with the models and methods in this unit will give them the conceptual understanding they need to understand how to correctly place the digits in the quotient and obtain the correct answer.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

Zeros In Quotients Cases involving 0 in division may require special attention.

- Case 1: a 0 in the dividend
- Case 2: a 0 in a remainder
- Case 3: a 0 in the quotient

Estimating Quotients

Lesson 8

As with addition, subtraction, and multiplication, this unit presents estimation as a way to verify the reasonableness of quotients. Students learn two methods. In one method, the quotient is rounded and multiplied by the divisor to determine if the product is close to the dividend. In the other method, the dividend is rounded and divided by the divisor to determine if the estimated quotient is close to the actual quotient. When students use the second method, they round the dividend to a number that is close to the actual dividend, but is easy to divide by the divisor. For example, to estimate 5,568 ÷ 6, students round 5,568 to 5,400, because 54 ÷ 6 = 9. In this case, the estimated quotient is 5,400 ÷ 6 = 900.

Problem Solving

Lessons 9 10

Problem Solving Plan In *Math Expressions* a research-based problem-solving approach that focuses on problem types is used.

- Interpret the problem
- Represent the situation
- Solve the problem
- Check that the answer makes sense.

Remainders When students solve problems involving division, they learn that the remainder can affect the solution to the problem. In Lesson 9, they encounter the following five types of word problems:

A. A remainder that is not part of the question *Thomas has one 9-foot pine board. He needs to make 4-foot shelves for his books. How many shelves can he cut?* The 1-foot board cannot be used to make another 4-foot shelf. He can make 2 shelves, and the remainder is ignored.

B. A remainder that causes the answer to be rounded up *Nine students are going on a field trip. Parents have offered to drive. If each parent can drive 4 students, how many parents need to drive?* Two cars are not enough to transport 9 students. They need 3 cars.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Remainders in Word Problems

In problem situations, students must interpret and use remainders with respect to context. For example, what is the smallest number of busses that can carry 250 students, if each bus holds 36 students? The whole number quotient in this case is 6 and the remainder is 34; the equation 250 = 6 × 36 + 34 expresses this result and corresponds to a picture in which 6 busses are completely filled while a seventh bus carries 34 students. Notice that the answer to the stated question (7) differs from the whole number quotient.

On the other hand, suppose 250 pencils were distributed among 36 students, with each student receiving the same number of pencils. What is the largest number of pencils each student could have received? In this case, the answer to the stated question (6) is the same as the whole number quotient. If the problem had said that the teacher got the remaining pencils and asked how many pencils the teacher got, then the remainder would have been the answer to the problem.

C. A fraction remainder *One Monday Kim brought 9 apples to school. She shared them equally among herself and 3 friends. How many apples did each person get?* Kim and her friends can each have 2 whole apples. They can divide the ninth apple into 4 equal parts, so that each person can have $2\frac{1}{4}$ apples.

D. A decimal remainder *Raul bought 4 toy cars for $9.00. Each car costs the same amount. How much did each car cost?* Dollars, like apples, can be divided. We know that there are 4 quarters in $1, and we write $\frac{1}{4}$ dollar as $0.25, so each toy car cost $2.25.

E. Remainder only *Nine students have signed up to run a relay race. If each relay team can have 4 runners, how many students cannot run in the race?* All of the teams have to be the same size. Any extra people cannot participate in the race. In this case, there can be only 2 teams of 4 each. The remainder represents the extra person who cannot participate.

Part of students developing fluency in their ability to use division to solve problems is to gain the understanding of these types of problems so that they can correctly identify the solution after they have divided.

While students are not required to write the remainder as a fraction or decimal, they are included in the class discussion so students will see the big picture.

Choosing the Operation Students are presented with a variety of one-step and multistep word problems. They are encouraged to attend to the structure of the problem to determine the operation needed to solve it. Students review situation and solution equations. They develop an understanding that, although the situation equation may indicate a certain operation, because of the number that is unknown, the solution equation may indicate that they need to use a different operation to solve the problem.

from FROM THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Word Problems Fourth graders extend problem solving to multistep word problems using the four operations posed with whole numbers. The same limitations discussed for two-step problems concerning representing such problems using equations apply here. Some problems might easily be represented with a single equation, and others will be more sensibly represented by more than one equation or a diagram and one or more equations.

Focus on Mathematical Practices

Lesson 11

The standards for Mathematical Practice are included in every lesson of this unit. However, there is an additional lesson that focuses on all eight Mathematical Practices. In this lesson, students use what they know about dividing whole numbers to solve problems involving a class trip to an amusement park.

Multiplication and Division Basic Facts Fluency

At this grade level students should be able to recall multiplication and division facts. If some students are still struggling with basic multiplication and division, you can use the diagnostic quizzes in the Teacher's Resource Book (M57 and M58) to assess their needs. Follow-up practice sheets are also provided. These practice sheets are structured so students can focus on a small group of multiplication and division facts on one sheet. There are also blank multiplication tables and scrambled multiplication tables to help students to develop instant recall.