Getting Ready to Teach Unit 2

Learning Path in the Common Core Standards

In previous grades, students learned basic facts and used these facts along with patterns, place value, and modeling to multiply one-digit numbers by multiples of 10. Unit 2 broadens and deepens students' experiences with multiplication to include multiplying numbers through thousands by one-digit numbers and finding the product of two two-digit numbers.

In this unit, students model the concepts of arrays, single-digit multiplication, place value, and area. The activities in this unit help students gain a conceptual understanding of multidigit multiplication. Students are expected to apply their understanding of multidigit multiplication to numeric calculations and real world problem solving situations, including multistep problems.

Math Expressions VOCABULARY

As you teach the unit, emphasize understanding of these terms.

- Place Value Sections Method
- Expanded Notation Method
- Algebraic Notation Method
- Shortcut Method

See the Teacher Glossary

Help Students Avoid Common Errors

Math Expressions gives students opportunities to analyze and correct errors, explaining why the reasoning was flawed.

In this unit, we use Puzzled Penguin to show typical errors that students make. Students enjoy explaining Puzzled Penguin's error and teaching Puzzled Penguin the correct way to multiply whole numbers. The following common errors are presented to the students as letters from Puzzled Penguin and as problems in the Teacher Edition that were solved incorrectly by Puzzled Penguin.

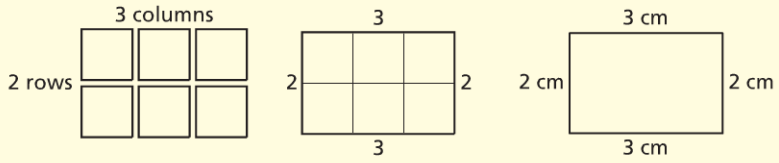
- **Lesson 2:** Using incorrect operations when factoring
- **Lesson 11:** When using new groups in multiplication, multiplying by the new groups rather than adding
- **Lesson 14:** Incorrectly choosing to overestimate or underestimate to solve a real world problem
- **Lesson 17:** Incorrectly rounding to estimate a product

In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As a part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.

Multiplying with Multiples of 10, 100, and 1000

Lessons 1 2 3

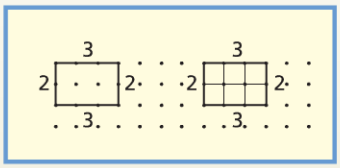
Array and Area Models In Grade 3, students modeled multiplication using array and area models. The models below show that a rectangle is made up of pushed-together squares of an array. All of these models show that $2 \times 3 = 6$. The models also illustrate that a rectangular model can be used to show equal groups as well as area. When working with the rectangular model with the small squares, students can visualize equal groups. When working with the rectangular model with no squares, students can visualize area.



Using these models is useful for students because it helps them utilize an idea with which they are familiar and expand it to represent more unfamiliar and complex concepts. This unit builds on the concept of area as pushed-together arrays of squares to help students understand multidigit multiplication.

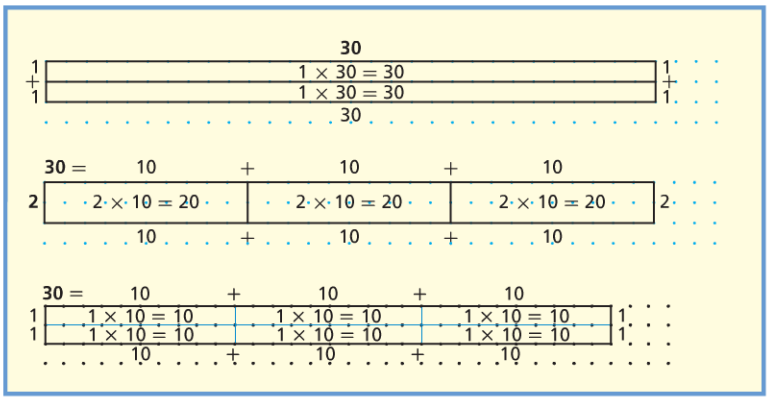
Multiplication and Place Value The exploration of multiplication begins by connecting students' understanding of place value to the concept of multiplication. Lessons on place value in Unit 1 provided visual support for the students' understanding that a 3 in the hundreds place does not stand for 3 ones, but actually represents 3 hundreds. It also helps prepare the way for the understanding of place value needed to conceptualize multidigit multiplication.

To multiply multiples of tens, students use dot drawings. When using dot drawings to model place value concepts, students counted the dots. In this unit, students find the number of unit "spaces" between the dots in order to find total areas of rectangles. The drawing below shows that $2 \times 3 = 6$ because there are 6 unit spaces enclosed in the drawing.



Multiplying a One-Digit Number by a Multiple of 10, 100, or 1000

In the multiplication models presented in the unit, little plus signs (+) every 10 spaces horizontally and vertically make it easier to count the units. The drawing below shows representations for the multiplication 2 × 30.

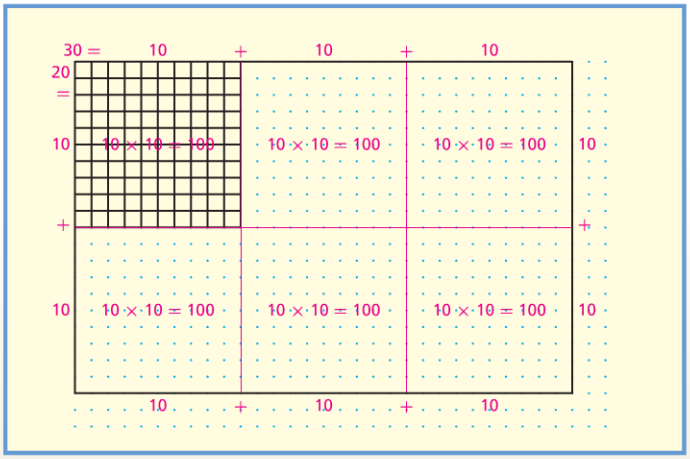


The drawings connect a visual representation to:

$$2 \times 30 = (2 \times 1) \times (3 \times 10)$$
$$= (2 \times 3) \times (1 \times 10)$$
$$= 6 \times 10 = 60$$

Multiplying Two Multiples of 10, 100, or 1,000

This drawing shows a representation for the multiplication 20 × 30. The drawing has all the squares visible in the first section to emphasize that the rectangular models can be used for both equal groups and area problems.



The drawing connects a visual representation to:

$$20 \times 30 = (2 \times 10) \times (3 \times 10)$$
$$= (2 \times 3) \times (10 \times 10)$$
$$= 6 \times 100$$
$$= 600$$

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

Multiplying by Multiples of 10, 100, and 1000 One component of understanding general methods for multiplication is understanding how to compute products of one-digit numbers and multiples of 10, 100, and 1000.

Patterns in Products In this unit, students also learn how to move from the pictorial to the abstract as they look at patterns of products that involve multiples of 10, 100, and 1,000. Students examine groups of related multiplications such as:

A	B	C	D
6 × 3	6 × 1 × 3 × 1	18 × 1	18
6 × 30	6 × 1 × 3 × 10	18 × 10	180
60 × 30	6 × 10 × 3 × 10	18 × 100	1,800

They rewrite each product so one factor is a power of ten. Then they see that the product is the product of the non-zero digits (in this case, 6 × 3), followed by the number of zeros in the power of 10. Area models help students understand why they can "count the zeros" when multiplying by tens and tens groups. These patterns can also help students understand how to estimate products by rounding and multiplying to check the accuracy of their calculations.

Understanding the connection between place value and multiplication forms the foundation for the multiplication work that students will experience throughout the unit. It provides the conceptual foundation necessary to understand the different multiplication methods students will learn.

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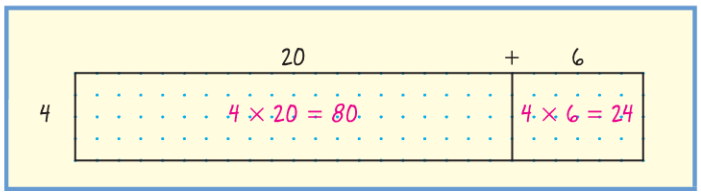
Patterns We can calculate 6 × 700 by calculating 6 × 7 and then shifting the result to the left two places (by placing two zeros at the end to show that these are hundreds) because 6 groups of 7 hundred is 6 × 7 hundreds, which is 42 hundreds, or 4,200. Students can use this place value reasoning, which can also be supported with diagrams of arrays or areas, as they develop and practice using the patterns in relationships among products such as 6 × 7, 6 × 70, 6 × 700, and 6 × 7000.

Multiplying by One-Digit Numbers

Lessons 4 6 7 8 9 10 16 17

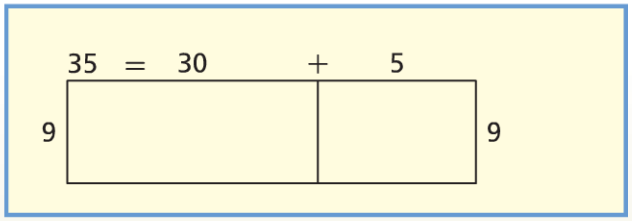
Using the Area Model As students begin to explore multiplying any two-digit number by a one-digit number, they expand the use of the area model they previously used when multiplying by multiples of 10. The first area models they use are dot drawings. In the area model below, the length represents 26, or 20 + 6. The width represents 4. The area of the first rectangle is the product of 4 × 20. The second product is 4 × 6. The areas, taken together, encompass the area of the whole rectangle, 4 × 26.



Thus, the model is a visual representation of:

$$\begin{aligned} 4 \times 26 &= 4 \times (20 + 6) \\ &= 4 \times 20 + 4 \times 6 \\ &= 80 + 24 \\ &= 104 \end{aligned}$$

As students gain a higher level of abstraction, they draw models without dots.



As with the dot drawing, this model shows that:

$$\begin{aligned} 9 \times 35 &= 9 \times (30 + 5) \\ &= 9 \times 30 + 9 \times 5 \\ &= 270 + 45 \\ &= 315 \end{aligned}$$

The area model is powerful because it can be generalized to any multidigit multiplication problem.

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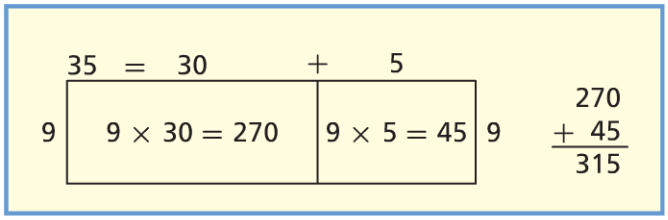
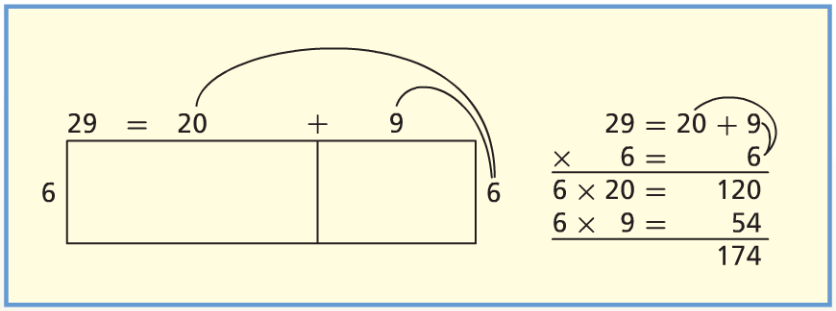
Modeling Multiplication As with addition and subtraction, students should use methods they understand and can explain. Visual representations such as area and array diagrams that students draw and connect to equations and other written numerical work are useful for this purpose.

Multiplication Methods This unit presents students with a variety of numerical multiplication methods. Each method can be represented by an area model. Additionally, in all of the methods, the value of the digit in each place of one number is multiplied by the value of the digit in each place of the other number. However, in each method, the manner in which the multiplication steps are recorded varies.

Place Value Sections Method Students draw a rectangle and add the areas of all the sections. The product equations are recorded inside the appropriate section of the rectangle. Then the products are added outside the rectangle. Notice that in this example, the first product, 270, is a result of multiplying 30 by 9. The second product, 45, is a result of multiplying 5 by 9. Students will come to know these products as partial products.



Expanded Notation Method In the Expanded Notation Method, the multiplication steps are recorded outside the rectangle. Arcs are used to ensure that every digit in one number is multiplied by every digit in the other number. Although students are introduced to using arcs in the one-digit multiplication lessons, it becomes a valuable strategy as students move into multidigit multiplication in which there are two or more digits in each factor.



Distributive Property The connection is made between the Distributive Property and the multiplication methods that students have learned. Students realize that if $a \times (b + c) = ab + ac$ and a two-digit number can be written as tens + ones, then it is possible to write, for example 5×43 as $5 \times (40 + 3)$, and thus find the sum of 5×40 and 5×3 to find the product.

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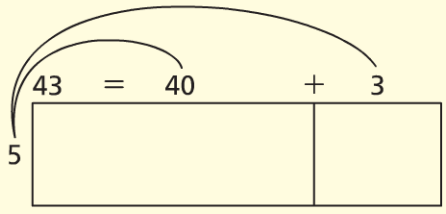
The Distributive Property Another part of understanding general base-ten methods for multidigit multiplication is understanding the role played by the distributive property. This allows numbers to be decomposed into base-ten units, products of the units to be computed, then combined. By decomposing the factors into like base-ten units and applying the distributive property, multiplication computations are reduced to single-digit multiplications and products of numbers with multiples of 10, of 100, and of 1000. Students can connect diagrams of areas or arrays to numerical work to develop understanding of general base-ten multiplication methods.

Algebraic Notation Method Students extend their understanding of the Distributive Property to using equations to record the multiplication steps. Notice that in this method, arcs are also used to keep track of the number of steps as well as to relate the numerical steps to the model.



$$\begin{aligned} 5 \bullet 43 &= 5 \bullet (40 + 3) \\ &= 5 \bullet 40 + 5 \bullet 3 \\ &= 200 + 15 \\ &= 215 \end{aligned}$$

Shortcut Method This is the common algorithm taught in most U.S. schools. Not all students are expected to gain proficiency using this method in this unit. Rather, it is preferred that students are capable of understanding multiplication using any of the methods presented. Just as in addition and subtraction, the new groups can be written above or below the problems.

Shortcut Method with New Groups Above

Method E:

Step 1

$$\begin{array}{r} {\scriptstyle 7\ \ } \\ 28 \\ \times\ 9 \\ \hline 2 \end{array}$$

Step 2

$$\begin{array}{r} {\scriptstyle 7\ \ } \\ 28 \\ \times\ 9 \\ \hline 252 \end{array}$$

Shortcut Method with New Groups Below

Method F:

Step 1

$$\begin{array}{r} 28 \\ \times\ 9 \\ \hline {\scriptstyle 7\ \ }2 \end{array}$$

Step 2

$$\begin{array}{r} 28 \\ \times\ 9 \\ \hline {\scriptstyle 7\ \ }252 \end{array}$$

Three- and Four-Digit Factors Multiplication in this unit is broadened to include multiplying one-digit numbers by three- and four-digit numbers. As students explore these problems, they gain the ability to generalize the methods that they learned. They observe that these same methods can be applied to three- and four-digit numbers. The following examples show the application of modeling and the Shortcut Method for recording multiplication to one-digit by three- and four-digit numbers.

Multiply 4 × 237

237 =	200	+ 30	+ 7	
4	4 × 200 = 800	4 × 30 = 120	4 × 7 = 28	4

800
120
+ 28
948

→

$$\begin{array}{r} {\scriptstyle 1\,2} \\ 237 \\ \times\ \ 4 \\ \hline 948 \end{array}$$

Multiply 9 × 6,435

6,435 =	6,000	+ 400	+ 30	+ 5	
9	9 × 6,000 = 54,000	9 × 400 = 3,600	9 × 30 = 270	9 × 5 = 45	9

54,000
3,600
270
+ 45
57,915

→

$$\begin{array}{r} {\scriptstyle 3\,3\,4} \\ 6{,}435 \\ \times\ \ \ 9 \\ \hline 57{,}915 \end{array}$$

Multiplying Two Two-Digit Numbers

Lessons 12 13 15 18

Connecting Two-Digit Multiplication with One-Digit Multiplication

As students continue to explore multiplication, they move to multiplying two two-digit numbers. Although the problems in these lessons become more complex, the work that students did with one-digit factors forms the foundation for the understandings they need to proceed to greater numbers. The model and the methods already presented are powerful because they can be generalized to multiplication with any number of digits. Thus, as students progress through the unit, they continue to expand the application of the models and procedures they have already learned.

Modeling and Methods The following examples show how the models and methods for which students have already developed fluency are extended to multiplication with greater numbers. The use of the arcs is expanded. Using color to code the arcs benefits students because it allows them to more carefully track and organize the steps in the multiplication.

Place Value Sections Method

$$\begin{aligned} 40 \times 60 &= 2{,}400 \\ 40 \times 7 &= 280 \\ 3 \times 60 &= 180 \\ 3 \times 7 &= + \ 21 \\ &\quad 2{,}881 \end{aligned}$$

Expanded Notation Method

$$\begin{array}{r} 67 \\ \times \ 43 \end{array} \quad \begin{array}{l} 60 + 7 \\ 40 + 3 \end{array}$$

$$\begin{aligned} 40 \times 60 &= 2{,}400 \\ 40 \times 7 &= 280 \\ 3 \times 60 &= 180 \\ 3 \times 7 &= + \ 21 \\ &\quad 2{,}881 \end{aligned}$$

Algebraic Notation Method

$$\begin{aligned} 43 \cdot 67 &= (40 + 3) \cdot (60 + 7) \\ &= 2{,}400 + 280 + 180 + 21 \\ &= 2{,}881 \end{aligned}$$

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

Two-Digit by Two-Digit Multiplication Computing products of two two-digit numbers requires using the distributive property several times when the factors are decomposed into base-ten units. For example,

$$\begin{aligned} 36 \times 94 &= (30 + 6) \times (90 + 4) \\ &= (30 + 6) \times 90 + \\ &\quad (30 + 6) \times 4 \\ &= (30 \times 90) + (6 \times 90) + \\ &\quad (30 \times 4) + (6 \times 4) \end{aligned}$$

Shortcut Method Notice that the new groups are written above the numbers. Often when multiplying two-digit numbers, it is necessary to write more than one set of new groups. Since there is usually more room above the problem than below, recording the new groups in this fashion is more efficient.

New Groups Above

Step 1	Step 2	Step 3	Step 4	Step 5
$\overset{2}{67} \times 43$	$\overset{2}{67} \times 43$	$\overset{2}{\overset{2}{67}} \times 43$	$\overset{2}{\overset{2}{67}} \times 43$	$\overset{2}{\overset{2}{67}} \times 43$
1	201	201	201	201
		8	268	+ 268
				2,881

Estimating Products

Lessons 5 14

In Lessons 5 and 14 students learn how to estimate products. Students learn that by rounding the factors and multiplying them, an approximate answer for a product can be attained. Once the factors have been rounded, students can apply their understanding of multiplying with multiples of ten. A rounded estimate helps students decide if their answers are reasonable. Students estimate products with one- and two-digit factors.

Problem Solving

Lesson 11

Problem Solving Plan In *Math Expressions* a research-based problem-solving approach that focuses on problem types is used.

- Interpret the problem.
- Represent the situation.
- Solve the problem.
- Check that the answer makes sense.

Hidden Questions In some problems, students must find the answer to a hidden question and use that answer to answer the question of the problem. We use the term *hidden questions* to make the conceptual point that students may need to answer these questions even if they do not appear in the problem.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Word Problems Fourth graders extend problem solving to multistep word problems using the four operations posed with whole numbers. The same limitations discussed for two-step problems concerning representing such problems using equations apply here. Some problems might easily be represented with a single equation, and others will be more sensibly represented by more than one equation or a diagram and one or more equations.

Too Much Information/Too Little Information Throughout the unit, real world situations are used as the context for problem solving situations, including problems that involve two or more different operations. Students are also expected to analyze problems to determine whether they have too little or too much information.

Focus on Mathematical Practices

Lesson 19

The standards for Mathematical Practice are included in every lesson of this unit. However, there is an additional lesson that focuses on all eight Mathematical Practices. In this lesson, students use what they know about multiplying whole numbers to play a game in which they design and build a successful city while adhering to a specific budget.

Multiplication and Division Basic Facts Fluency

At this grade level students should be able to recall multiplication and division facts. If some students are still struggling with basic multiplication and division, you can use the diagnostic quizzes in the Teacher's Resource Book (M57 and M58) to assess their needs. Follow-up practice sheets are also provided. These practice sheets are structured so students can focus on a small group of multiplication and division facts on one sheet. There are also blank multiplication tables and scrambled multiplication tables to help students to develop instant recall.