Getting Ready to Teach Unit 7

Learning Path in the Common Core Standards

In this unit, students study fraction concepts, beginning with unit fractions and what they represent. Students learn how non-unit fractions are built from unit fractions, and to count or add to find the value of a fraction. They model fractions in various ways as they compare fractions and find equivalent fractions.

Visual models and real world situations are used throughout the unit to illustrate important fraction concepts.

Math Expressions VOCABULARY

As you teach this unit, emphasize understanding of these terms.

- unit fraction
- equivalent fractions

See the Teacher Glossary.

Help Students Avoid Common Errors

Math Expressions gives students opportunities to analyze and correct errors, explaining why the reasoning was flawed.

In this unit, we use Puzzled Penguin to show typical errors that students make. Students enjoy teaching Puzzled Penguin the correct way, why this way is correct, and why Puzzled Penguin made the error. Common errors are presented in Puzzled Penguin features in the following lessons:

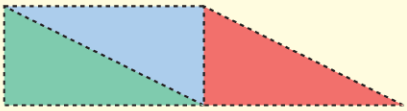
- **Lesson 5:** finding that two fractions with the same numerator are equal without looking at their denominators
- **Lesson 6:** naming an equivalent fraction for $\frac{1}{2}$ that does not name the same part of the whole

In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As a part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.

Fraction Concepts

Lessons 1 2 3

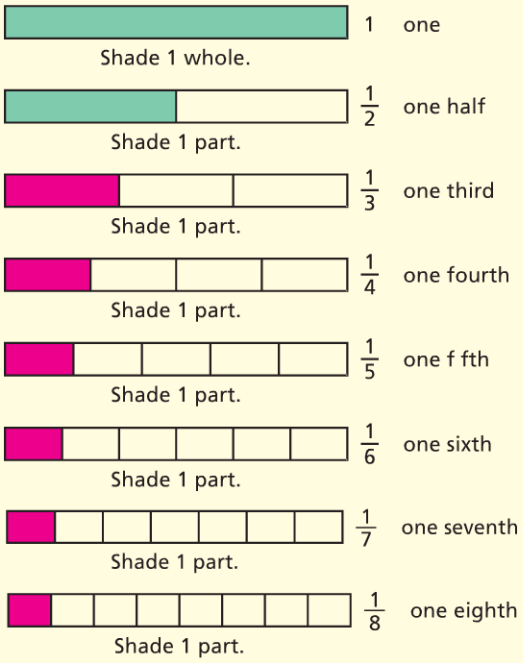
Fractions In Grade 3, students study fraction concepts in depth. They first use shapes to explore the meaning of a unit fraction, $\frac{1}{d}$, as the quantity formed by 1 part when a whole is partitioned into d equal parts. The work students have done with decomposing shapes gives them a foundation for this work with equal parts of a whole.

6. 

There are 3 equal parts in the whole shape.

The blue triangle is $\frac{1}{3}$ of the whole shape.

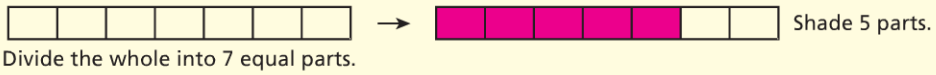
Fraction bars help students visualize the relationship between unit fractions with different denominators. The visual representation fosters the concept that a unit fraction with a smaller denominator is greater than a unit fraction with a larger denominator. For example, $\frac{1}{2}$ is greater than $\frac{1}{8}$.



from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS—FRACTIONS

The meaning of fractions Grade 3 students start with unit fractions (fractions with numerator 1), which are formed by partitioning a whole into equal parts and taking one part, e.g., if a whole is partitioned into 4 equal parts then each part is $\frac{1}{4}$ of the whole, and 4 copies of that part make the whole. Next, students build fractions from unit fractions, seeing the numerator 3 of $\frac{3}{4}$ as saying that $\frac{3}{4}$ is the quantity you get by putting 3 of the $\frac{1}{4}$'s together. They read any fraction this way, and in particular there is no need to introduce "proper fractions" and "improper fractions" initially; $\frac{5}{3}$ is the quantity you get by combining 5 parts together when the whole is divided into 3 equal parts.

Meaning of a Fraction Students learn that a fraction, $\frac{n}{d}$, is the quantity formed by n equal parts of size $\frac{1}{d}$. With this definition of a fraction, it is not necessary to differentiate between fractions less than 1 and fractions equal to or greater than 1. Conceptualizing fractions as being composed of unit fractions added together is like thinking of whole numbers as being composed of ones added together.

11. 

Divide the whole into 7 equal parts.

$\frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7}$ $\quad\quad$ $\frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} = \frac{5}{7}$

Just as $5 = 1 + 1 + 1 + 1 + 1$, $\frac{5}{7} = \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7}$. Some students find this analogy helpful in understanding the relationship between unit fractions and all fractions.

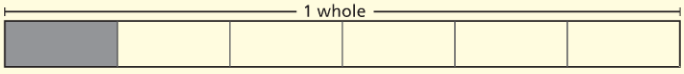
As students construct fractions from fraction bars, they see that they can count the number of equal parts in a fraction or they can add the unit fractions to find the value of that fraction.

Model Fractions Two representations, fraction bars and number lines, are used to model fractions. Students use both ways to model the same set of fractions and then compare the two.

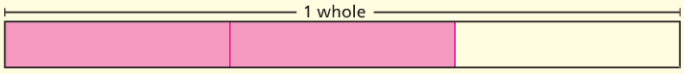
▶ Use Fraction Bars

Shade each fraction bar to show the fraction.
First, divide the fraction bar into the correct unit fractions.

1. $\frac{1}{6}$



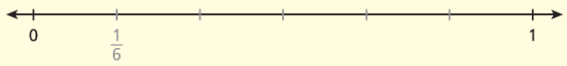
2. $\frac{2}{3}$



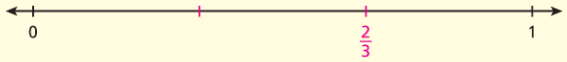
▶ Use Number Lines

Mark each number line to show the fraction.
First, divide the number line into the correct unit fractions.

7. $\frac{1}{6}$



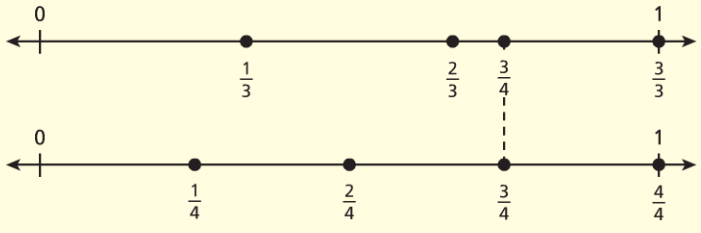
8. $\frac{2}{3}$



Locate Fractions on the Number Line As students work with fractions on the number line, they see that each fraction is located at a point on the number line. Because fractions can be located on a number line, they must be numbers. Many students find this a compelling rationale for thinking of fractions as numbers.

Students are presented with various tasks dealing with locating numbers on the number line, including locating several unit fractions on the same number line and locating fractions greater than 1 on a number line. They learn to use a ruler to help locate fractions, to use their fingers to "mark off" unit fractions until they locate a particular fraction, and to use more than one number line. The key concept for students is to think of the interval from 0 to 1 as a whole, partition the whole into as many same size parts as the denominator, and then locate a particular fraction.

Tasks similar to the following may someday be on standardized tests: students are given the point for $\frac{1}{3}$ on a number line and asked to locate $\frac{3}{4}$. This task brings together concepts of the meaning of fractions and of locating fractions on a number line. To solve, students first locate $\frac{3}{3}$ or 1 on an equivalent number line, then use the distance from 0 to 1 to locate $\frac{1}{4}$, $\frac{2}{4}$, and $\frac{3}{4}$ on another equivalent number line, and finally transfer the point for $\frac{3}{4}$ back to the original number line.



Compare Fractions

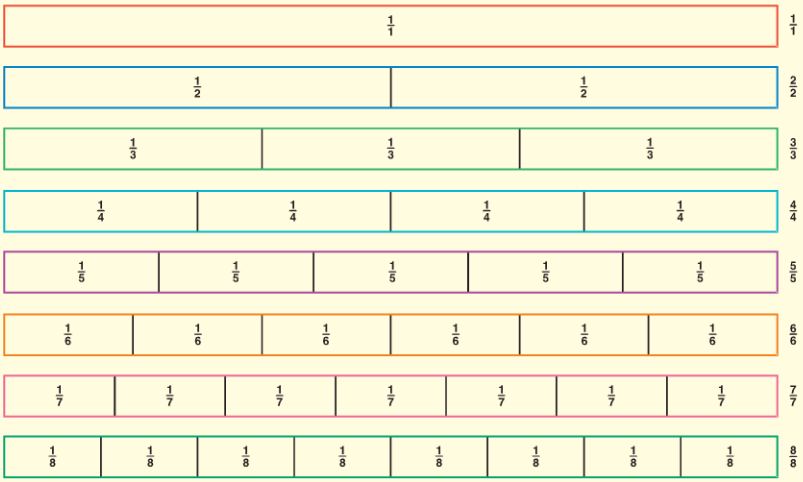
Lessons 4 5

In Grade 3, students use reasoning and number sense to compare fractions. In Lesson 4, they compare unit fractions and in Lesson 5, they compare non-unit fractions. As with whole numbers, the symbols >, =, and < are used to record the results of the comparisons.

Unit Fractions When students shaded fraction bars in Lesson 1 to build unit fractions, they were taking the first step toward comparing unit fractions.

Now students examine fraction bars from 1 to $\frac{1}{8}$, observe differences and patterns in the unit fractions, and use their observations to decide that they can compare the denominators of two unit fractions to find which is the smaller fraction and which is the greater fraction.

The fraction bars are made up of unit fractions. Look for patterns.



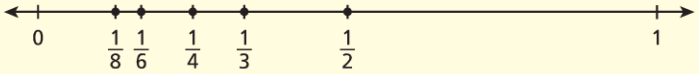
With the help of this visual representation, students learn that the unit fraction with the greater denominator will be the smaller fraction and the unit fraction with the smaller denominator will be the greater fraction.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS—FRACTIONS

Comparing fractions Students also see that for unit fractions, the one with the larger denominator is smaller, by reasoning, for example, that in order for more (identical) pieces to make the same whole, the pieces must be smaller. From this they reason that for fractions that have the same numerator, the fraction with the smaller denominator is greater. For example, $\frac{2}{5} > \frac{2}{7}$, because $\frac{1}{7} < \frac{1}{5}$, so 2 lengths of $\frac{1}{7}$ is less than 2 lengths of $\frac{1}{5}$.

Students also examine unit fractions on a number line and observe how the position on the number line relates to the denominator of the fraction. They see that smaller fractions lie farther to the left on the number line. Some students may even notice that $\frac{1}{2}$ is the greatest unit fraction and that the distances between the unit fractions grow smaller as you move to the left on the number line.

The number line shows unit fractions. Look for patterns in the number line.



Non-Unit Fractions Students use fraction circle parts for fourths, sixths, and eighths to construct and compare fractions with denominators of 4, 6, and 8. Building on their understanding of unit fractions, they see that they can compare fractions that have the same numerator or fractions that have the same denominator by using reasoning about the size of the parts. It is important to emphasize that in order to compare fractions, the wholes for which the fractions are a part must be the same size.

The students will find that when two fractions have the same numerator, the fraction with the smaller denominator is the greater fraction and that when two fractions have the same denominator, the fraction with the greater numerator is the greater fraction.

Students practice comparing non-unit fractions and record the results using the comparison symbols: <, =, and >.

Compare. Use <, >, or =.

7. $\frac{2}{2} > \frac{2}{3}$	8. $\frac{1}{3} < \frac{5}{3}$	9. $\frac{3}{2} > \frac{3}{6}$	10. $\frac{5}{6} > \frac{4}{6}$
11. $\frac{4}{6} < \frac{5}{6}$	12. $\frac{3}{4} > \frac{3}{8}$	13. $\frac{6}{3} > \frac{5}{3}$	14. $\frac{8}{4} > \frac{8}{7}$
15. $\frac{5}{6} < \frac{5}{3}$	16. $\frac{8}{5} < \frac{12}{5}$	17. $\frac{6}{5} < \frac{6}{4}$	18. $\frac{2}{2} = \frac{4}{4}$
19. $\frac{5}{8} > \frac{3}{8}$	20. $\frac{7}{3} > \frac{7}{6}$	21. $\frac{7}{8} > \frac{3}{8}$	22. $\frac{9}{4} > \frac{9}{8}$

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS—FRACTIONS

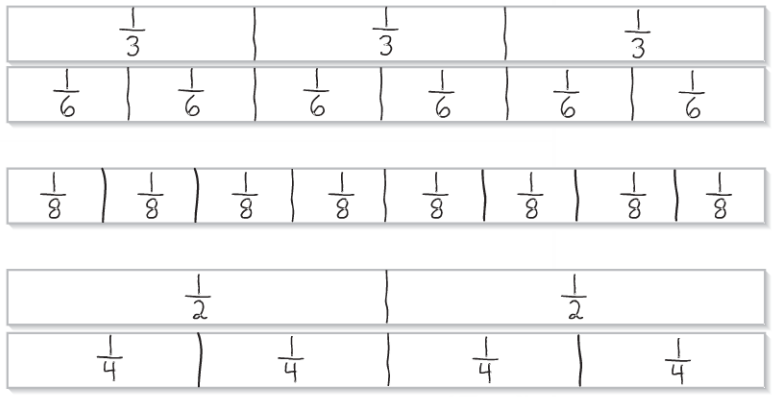
Comparing fractions Previously, in Grade 2, students compared lengths using a standard measurement unit. In Grade 3 they build on this idea to compare fractions with the same denominator. They see that for fractions that have the same denominator, the underlying unit fractions are the same size, so the fraction with the greater numerator is greater because it is made of more unit fractions. For example, the segment from 0 to $\frac{3}{4}$ is shorter than the segment from 0 to $\frac{5}{4}$ because it measures 3 units of $\frac{1}{4}$ as opposed to 5 units of $\frac{1}{4}$. Therefore, $\frac{3}{4} < \frac{5}{4}$.

Equivalent Fractions

Lessons 6 7

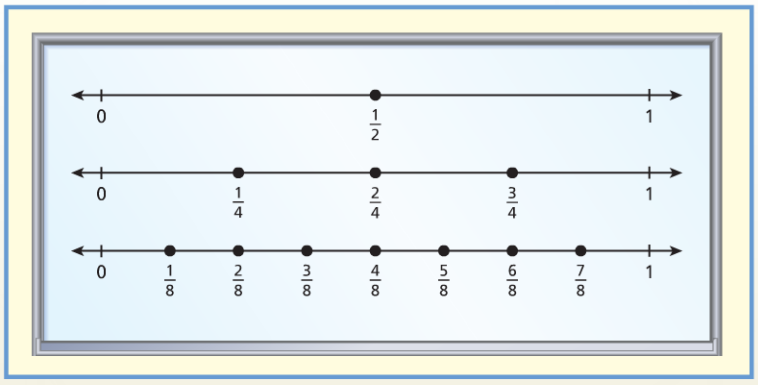
Fraction Strips Students fold same size strips of paper that represent 1 into different fractions: halves, fourths, eighths, thirds, and sixths. The beginning of this activity is another opportunity to emphasize that only fractions that have the same whole can be compared.

After students fold and label the fraction strips, they use them to find fractions that are the same length and they learn that these equal fractions are called *equivalent fractions*. Their collection of folded strips should look something like this.



Students notice or measure to see that $\frac{1}{2}$, $\frac{2}{4}$, $\frac{4}{8}$, and $\frac{3}{6}$ have the same length and are equivalent fractions.

Number Lines A similar activity using number lines leads to the same results. This activity is an opportunity to reiterate that fractions are numbers, since they can be represented on the number line.



from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS—FRACTIONS

Equivalent fractions Grade 3 students do some preliminary reasoning about equivalent fractions, in preparation for work in Grade 4. As students experiment on number line diagrams they discover that many fractions label the same point on the number line, and are therefore equal; that is, they are *equivalent fractions*. For example, the fraction $\frac{1}{2}$ is equal to $\frac{2}{4}$ and also to $\frac{3}{6}$. Students can also use fraction strips to see fraction equivalence.

Word Problems Involving Fractions

Lesson 8

Solve Problems As with all mathematical topics students have covered this year, it is important for them to see that they can use the fraction concepts they have just learned to solve real world problems. Fractions are found in measurement problems, including hiking, woodworking, and cooking. Continue to use **Solve and Discuss** and encourage questioning. Be sure that students respond with clear explanations. They may use any of the models for fractions from this unit to help them illustrate a point.

Solve. Draw diagrams or number lines if you need to.

1. The shelves in Roger's bookcase are $\frac{7}{8}$ yard long. Latanya's bookcase has shelves that are $\frac{5}{8}$ yard long. Whose bookcase has longer shelves? How do you know?
   Roger's bookcase; When you compare $\frac{7}{8}$ and $\frac{5}{8}$, $\frac{7}{8} > \frac{5}{8}$.

2. Rosa buys $\frac{3}{4}$ pound of cheddar cheese. Lucy buys $\frac{3}{8}$ pound of goat cheese. Who buys more cheese? Explain your answer.
   Rosa; When you compare $\frac{3}{4}$ and $\frac{3}{8}$, $\frac{3}{4} > \frac{3}{8}$.

Focus on Mathematical Practices

Lesson 9

The Standards for Mathematical Practice are included in every lesson of this unit. However, the last lesson in every unit focuses on all eight Mathematical Practices. In this lesson, students apply what they have learned about fractions to what they have learned about geometry to solve problems about paper folding and designs.