Getting Ready to Teach Unit 6

Math Expressions VOCABULARY

As you teach this unit, emphasize understanding of these terms.

- triangle
- quadrilateral
- area

See the Teacher Glossary.

Learning Path in the Common Core Standards

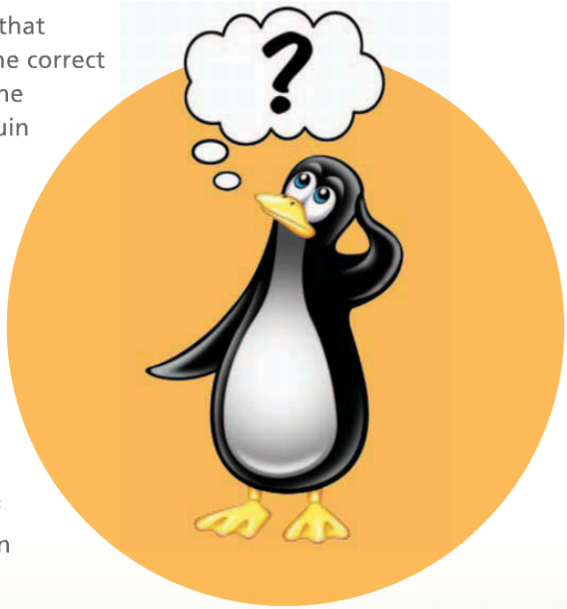
In this unit, students study the attributes of triangles, quadrilaterals, and other polygons. They find perimeter and area of various shapes and delve deeply into concepts of area of rectangular figures. They explore the relationship between perimeter and area by investigating rectangles with the same perimeter and different areas and rectangles with the same area and different perimeters.

Visual models and real world situations are used throughout the unit to illustrate important fraction concepts.

Help Students Avoid Common Errors

Math Expressions gives students opportunities to analyze and correct errors, explaining why the reasoning was flawed.

In this unit, we use Puzzled Penguin to show typical errors that students make. Students enjoy teaching Puzzled Penguin the correct way, why this way is correct, and why the error is wrong. The common errors are presented as letters from Puzzled Penguin to the students:



- **Lesson 4:** drawing a hexagon to represent a quadrilateral with parallel sides that is not a rectangle, square, or rhombus
- **Lesson 8:** decomposing a figure correctly into three rectangles, but only labeling and finding the areas of two of the rectangles

In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As a part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.

Geometric Concepts

Lessons 1 2 3 4

The Grade 3 Common Core State Standards ask that students reason with shapes and their attributes in two ways.

1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.

In studying shape and structure across the grades, students progressively become able to see and reason about properties of shape and to organize shapes into categories based on properties. Grade 3 students reason about subcategories of shapes (for example, rhombuses, rectangles, and others) and recognize that these shapes may share attributes that define a larger category (for example, quadrilaterals).

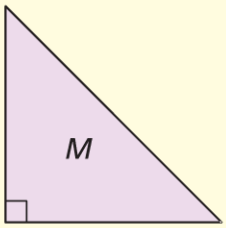
Work with Triangles and Polygons

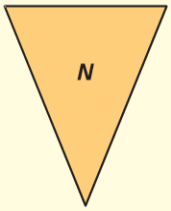
Lesson 1

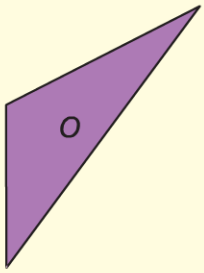
Lesson 1 establishes a foundation for the rest of the work in this unit by providing the vocabulary needed to discuss geometric concepts, explaining ways to classify and name various polygons, and showing how some figures can be composed of or decomposed into triangles. You may want to use more than one class period for this lesson depending on how much is review for your students.

Classify Angles Students learn that a right angle forms a square corner and they compare other angles to right angles as angles that are larger than or smaller than a right angle. As students draw angles that are the same as or larger than or smaller than a right angle, they are building understanding of angle relationships that will help them use protractors successfully in later years.

Classify Triangles Building on this understanding, students learn to classify triangles as triangles with a right angle (right triangles), with three angles smaller than a right angle (acute triangles), and with an angle larger than a right angle (obtuse triangles). Students also learn to classify triangles by the lengths of the triangle sides: 3 sides of equal length (equilateral or isosceles), 2 sides of equal length (isosceles), and no sides of equal length (scalene). Students describe triangles both by angles and sides but do not use the technical terms. They understand that all closed shapes with three sides are triangles.





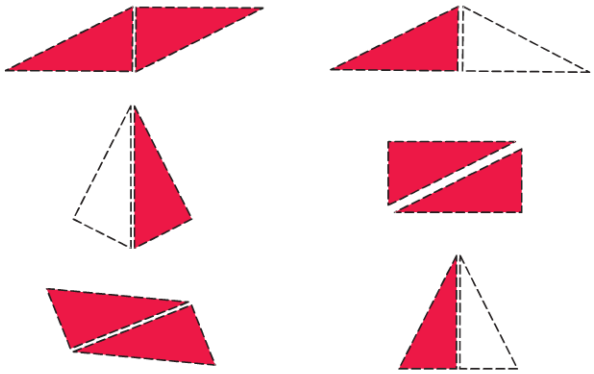


10. Triangle ___O___ has 1 angle larger than a right angle and has ___0___ sides of equal length.

11. Triangle ___M___ has 1 right angle and has ___2___ sides of equal length.

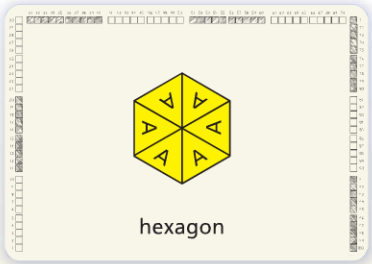
12. Triangle ___N___ has 3 angles smaller than a right angle and has ___2___ sides of equal length.

Quadrilaterals After working with triangles, students explore quadrilaterals by composing quadrilaterals from two congruent triangles. They use each type of triangle based on angle measure. They discover that two triangles usually can be put together to form different quadrilaterals, but that in the case of two right triangles, they can also form larger triangles. This hands-on activity helps students develop spatial sense as they see how quadrilaterals can be composed from triangles.



Polygons Students learn a definition of a polygon and that polygons may be concave or convex. They see how polygons are named for the number of sides they have and learn the names for several common polygons: triangle, quadrilateral, pentagon, hexagon, octagon, and decagon.

Cutting up polygons to see how they can be composed of triangles and using triangles to form polygons provides students with another hands-on experience that develops spatial sense. Such activities establish a conceptual base for area that helps students understand area formulas.



Lessons 2 3 4

Quadrilaterals

Attributes of Quadrilaterals The terms *parallel* and *perpendicular* are used to describe the sides of quadrilaterals. As students observe how the sides of quadrilaterals are related, they learn to name a quadrilateral based on these relationships.

Quadrilateral Names and Attributes

Quadrilateral	a polygon with four sides
Parallelogram	a quadrilateral with both pairs of opposite sides parallel
Rectangle	a parallelogram with four right angles
Square	a rectangle with all sides the same length
Rhombus	a parallelogram with all sides the same length
Trapezoid	a quadrilateral with one pair of opposite sides parallel

Students also learn that a quadrilateral may have several names. For example, a rhombus is a parallelogram and a quadrilateral. Although students are not taught to use the most specific name for a quadrilateral, many will see that the more specific a name is, the more information it communicates about the shape of the quadrilateral.

Use as many words below as possible to describe each figure.

quadrilateral	parallelogram	rectangle	square

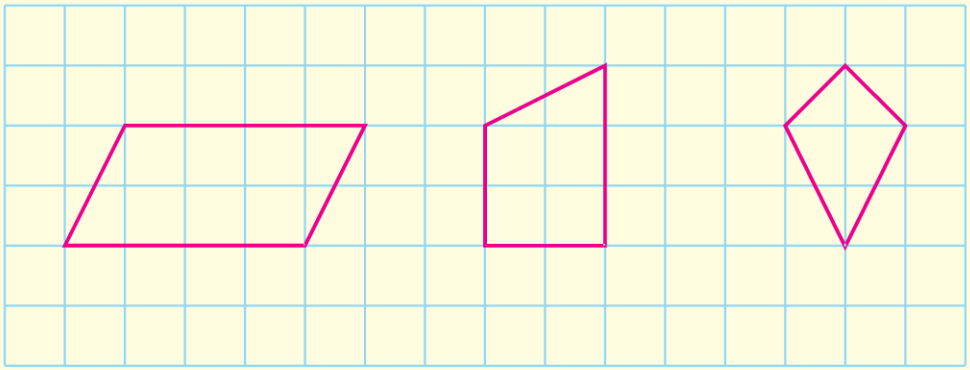
9. 

quadrilateral
parallelogram
rectangle

Draw Quadrilaterals Drawing quadrilaterals helps students clarify the relationships among the different quadrilaterals. This increases their knowledge of geometric shapes and provides support for their discussions of attributes and properties of geometric shapes.

Before drawing the shapes on the grids in the Student Activity Book, students describe what they know about each shape. Drawing more than one example of each kind of shape helps students recognize that the name of a shape simply describes the attributes of that shape.

11. Draw three different quadrilaterals that are not squares, rectangles, or rhombuses. Drawings will vary.

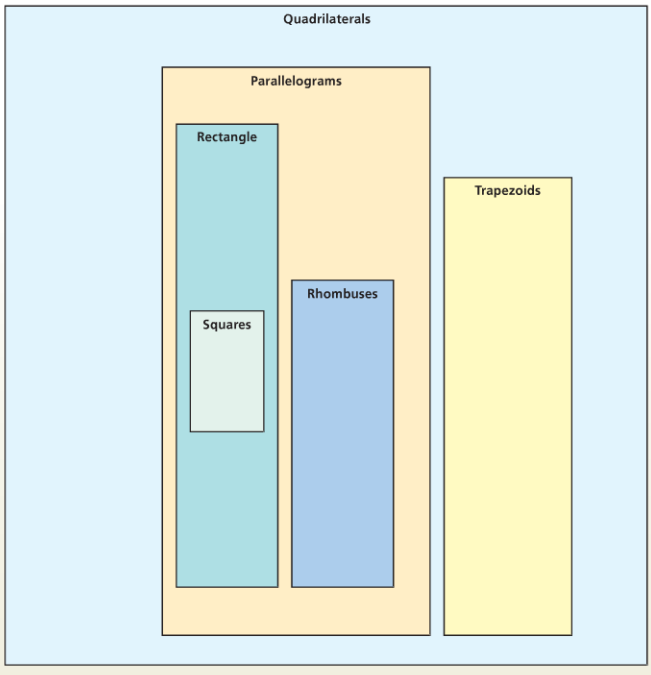


Students also draw quadrilaterals to match given descriptions. This activity requires them to take into consideration more than one attribute at a time.

Classify Quadrilaterals Properties of shapes and reasoning about these properties are important in the Common Core State Standards. Among the properties that third-graders will use to analyze and classify quadrilaterals are:

- sides of the same length or not (all or some)
- sides parallel or not (all or some)
- angles: right angle, smaller or larger than a right angle

Students use a category diagram to help them classify a set of quadrilaterals that they cut out from the Student Activity Book.

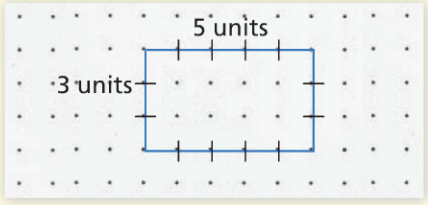


Perimeter and Area

Lessons 5 6 7 9

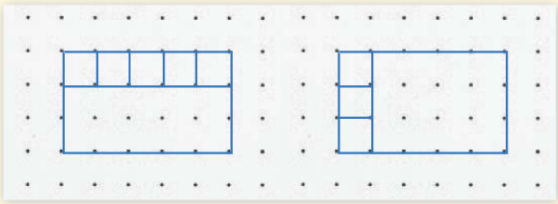
The concepts of perimeter and area bring together the domains of Geometry and Measurement and Data. The Geometric measurement standards for Grade 3 focus on area, from asking students to recognize area as an attribute of plane figures to guiding them to develop ways to measure area—tile with unit squares, see that the number of unit squares in an area is the same as the product of the side lengths, and recognize that an area model represents the Distributive Property of Multiplication over Addition. Although the main thrust of these lessons is area, students also investigate perimeter and they write equations for area and perimeter.

Tiling with Unit Squares Rectangular area models were used in Unit 3 to help students understand what happens when two factors are multiplied. They saw then that the product is the number of unit squares that fill a rectangle. This unit begins by re-establishing this important concept. Students construct rectangles on their MathBoards and determine the area by tiling and counting and by combinations of multiplying and adding. Some students may remember that they can multiply the side lengths.

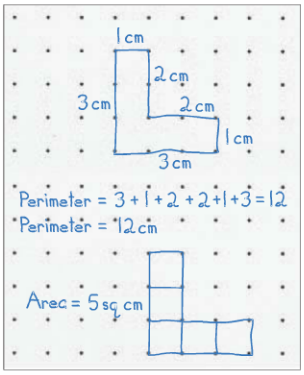


1 row is 5 sq cm.
3 rows are 3 × 5 sq cm, or 15 sq cm.

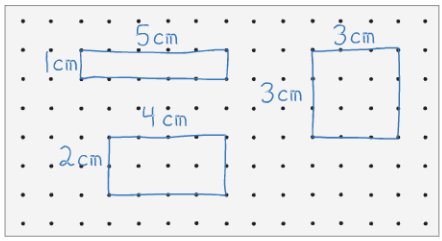
1 column is 3 sq cm.
5 columns are 5 × 3 sq cm, or 15 sq cm.



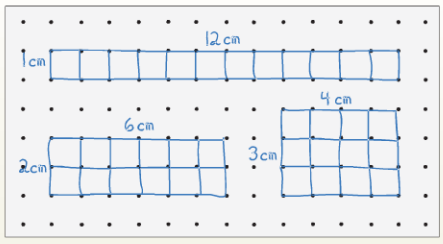
Perimeter and Area Although students studied perimeter as an attribute of plane figures in Grade 2, it is important that they understand how these two attributes are different. Exercises that ask students to find both measures will help you see whether students can differentiate perimeter and area. Emphasize that different units are used to describe each attribute: units of length for perimeter and square units for area.



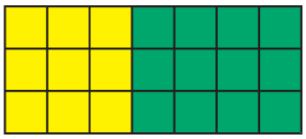
In Lesson 7, students relate perimeter and area as they draw on a dot array all the possible rectangles with a given perimeter with whole unit side lengths. Then they find the area of the rectangles. They observe for a given perimeter, the longest, skinniest rectangle has the least area and the most "square-like" rectangle has the greatest area.



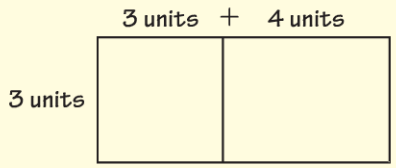
Students also draw all the possible rectangles with a given area with whole unit side lengths and find the perimeters. They observe that for a given area, the longest, skinniest rectangle has the greatest perimeter and the most "square-like" rectangle has the least perimeter.



Distributive Property Two different colors of self-stick notes are used as improvised units to construct a rectangle.



As the class members explore how to find the area of the rectangle, they discover the Distributive Property of Multiplication over Addition, although they do not use that term. They write an equation that represents the area of the rectangle.



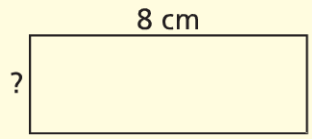
$3 \times 7 = (3 \times 3) + (3 \times 4)$

Students also describe and draw rectangles that represent an equation, such as $6 \times 9 = (6 \times 5) + (6 \times 4)$.

Unknown Side Lengths When students become more familiar with finding the area of a rectangle, they are asked to use what they know about the inverse relationship of addition and subtraction to find the length of a side given the length of the other side and the perimeter and what they know about the inverse relationship of multiplication and division to find the length of a side given the length of the other side and the area. Be sure always to refer to the unknown side length, and not the missing side length. The side is not missing because it is part of the rectangle; its length is what is not known.

Find the unknown side length in each diagram.

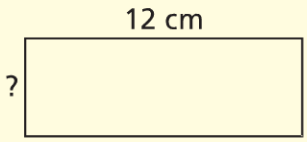
14.



Area = 72 sq cm

9 cm

15.



Perimeter = 38 cm

7 cm

Word Problems To help them appreciate that both perimeter and area are practical real world mathematical applications, students have plenty of practice solving word problems about realistic situations involving perimeter and area, including unknown side lengths. Since there are usually several ways to solve these problems, using **Solve and Discuss** helps students share ideas and learn from each. Encourage questioning and ask students to respond with clear explanations, using drawings to help support points they make.

Solve. Circle whether you need to find a perimeter, an area, or an unknown side length. Draw a diagram to represent each situation.

1. The dimensions of a rectangular picture frame are 9 inches and 6 inches. What is the area of a picture that would fit in the frame?

   Perimeter (Area) Side Length

   54 square inches

2. A garden has the shape of a regular hexagon. Each side of the garden is 5 feet long. How much fence is needed to go around the garden?

   (Perimeter) Area Side Length

   30 feet

3. The length of a water slide is 9 yards. The slide is 2 yards wide. How much of the surface of the slide must be covered with water?

   Perimeter (Area) Side Length

   18 square yards

4. Mr. Schmidt is installing 32 cubbies in the hallway. He puts 8 cubbies in each row. How many rows of cubbies can he make?

   Perimeter Area (Side Length)

   4 rows

Lesson 8

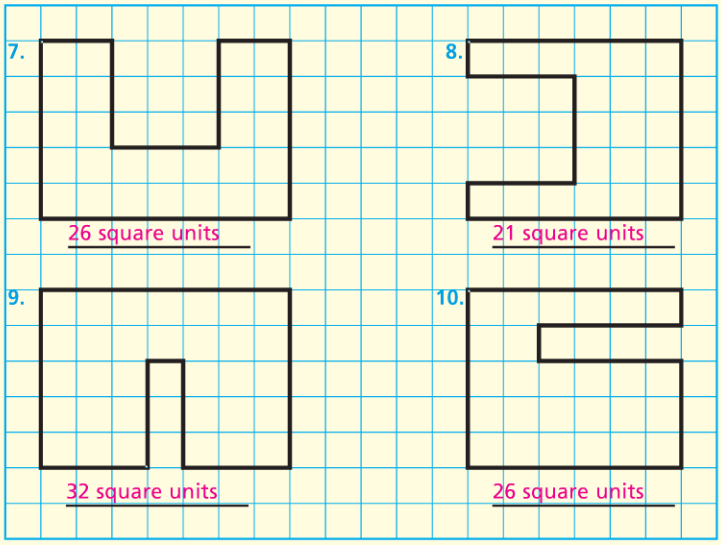
Rectilinear Figures

Additive Nature of Area Although students have worked with finding the area of rectangles, not all figures for which people need to find area are perfect rectangles. Because area is additive, it is possible to decompose or subdivide many figures into rectangles whose area can be determined and then add the areas of those rectangles to find the area of the figure. Such figures are sometimes called rectilinear figures, although the word *rectilinear* simply means "being made of straight lines."

Students do some investigative work with L-shaped figures on their MathBoards to help them see that area is additive. They see that adding the areas of two parts of a figure gives the same area as counting all the unit squares in the figure.

To find the area of such rectilinear figures, students are asked to look for rectangles within the figure. For some figures, students will need to find more than two rectangles. Watch for students who may form overlapping rectangles, as they will find too large an area for the figure and for students who do not completely cover the figures as they will find too small an area for the figure. You might tell students to watch for gaps and overlaps as they work.

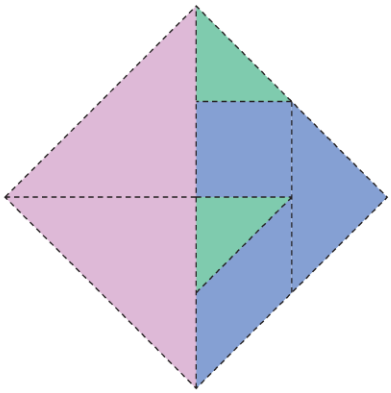
Decompose each figure into rectangles. Then find the area of the figure.



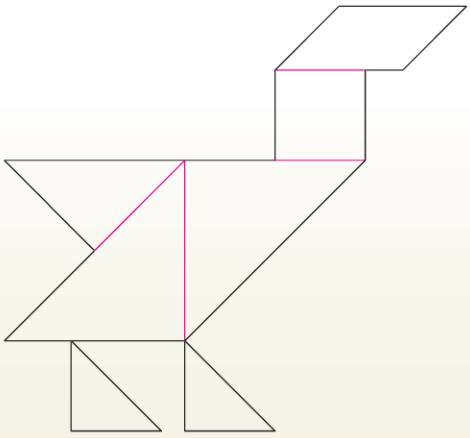
Lesson 10

Tangrams and Area

Tangrams are a popular puzzle that originated in China. The object of the puzzle is to create various figures from the set of seven shapes that make up the original tangram square. The *Math Expressions* tangrams are built on a 1-square inch grid, so that the areas of all the shapes can be expressed in square inches. Each purple triangle has an area of 2 square inches. Each blue shape (the square, the triangle, and the parallelogram) has an area of 1 square inch. Each green triangle has an area of $\frac{1}{2}$ square inch.

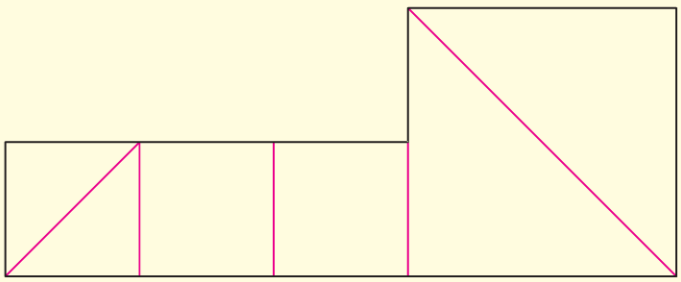


Build Figures After students cut out their tangram set, they use the shapes to build figures similar to the traditional tangram puzzles to become familiar with the shapes.



Use Tangram Pieces to Measure Area Once students are familiar with the shapes and can reproduce a given figure, they use two sets of tangram shapes and extra blue squares and green triangles to construct figures and calculate their areas. This helps to reinforce the concept that area is additive. Using the green triangles provides a preview of work in the next unit on fractions.

13.



What is the area of the figure?

7 square inches

Focus on Mathematical Practices

Lesson 11

The Standards for Mathematical Practice are included in every lesson of this unit. However, the last lesson in every unit focuses on all eight Mathematical Practices. In this lesson, students apply what they have learned about finding area and using appropriate tools to what they have learned about decomposing shapes into rectangles to solve problems about gardening.