Getting Ready to Teach Unit 4

Learning Path in the Common Core Standards

The lessons in this unit develop multidigit addition and subtraction methods that are meaningful and easily used by students. Place-value activities build understanding of the base ten numeration system and provide the foundation to understand the grouping and ungrouping concepts that students use to add and subtract. Students use drawings to show grouping and ungrouping, and then describe and discuss the process.

The activities in this unit help students gain practical understanding of addition and subtraction and the relationship between the two operations. They begin to see addition and subtraction as inverse operations and apply their knowledge of these concepts and skills to problem solving. Estimation provides students with methods to validate their answers.

Help Students Avoid Common Errors

Math Expressions gives students opportunities to analyze and correct errors, explaining why the reasoning was flawed.

In this unit, we use Puzzled Penguin to show typical errors that students make. Students enjoy teaching Puzzled Penguin the correct way, why this way is correct, and why Puzzled Penguin made the error. Common errors are presented in Puzzled Penguin features in the following lessons:

- **Lesson 6:** When rounding to the hundreds, rounding down although the number is closer to the greater hundred
- **Lesson 8:** Misaligning place values when adding multidigit numbers
- **Lesson 11:** When subtracting multidigit numbers, subtracting the lesser digit from the greater digit in each place rather than ungrouping

In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As a part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.

Math Expressions VOCABULARY

As you teach the unit, emphasize understanding of these terms.

- ten stick
- hundred box
- thousand bar
- Secret Code Cards
- Counting On strategy
- Make a Ten strategy
- place value drawings
- proof drawing
- Show All Totals method
- New Groups Below method
- New Groups Above method
- Make a Thousand strategy
- ungrouping
- Math Mountain

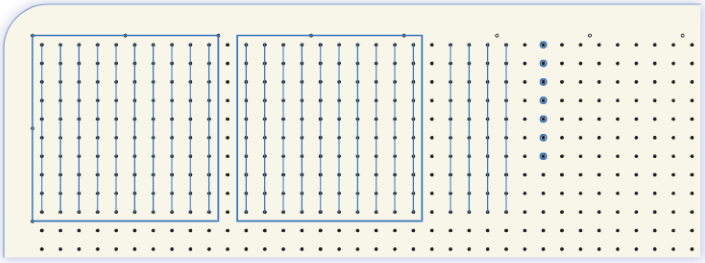
See the Teacher Glossary.

Place Value Concepts

Lessons 1 2 3 4

Place value drawings In *Math Expressions*, students use place value drawings to help them conceptualize numbers and understand the relative sizes of place values. Students begin by making these drawings on the dot-grid side of their MathBoards. They show ones by circling individual dots, tens by drawing lines through groups of ten dots, and hundreds by drawing squares around groups of 100 dots. The terms *ones*, *ten sticks*, and *hundred boxes* are used to describe the three representations. The drawing below represents the number 247. It shows:

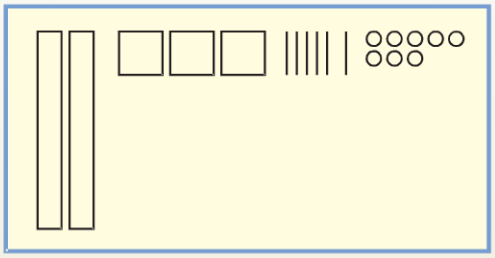
- 2 hundred boxes (2 squares that each contain 100 dots) = 200
- 4 ten sticks (4 line segments that each connect 10 dots) = 40
- 7 ones (7 circles that each contain 1 dot) = 7



The place value drawing is a beneficial model because it helps students visualize the magnitude of numbers. For example, in this model, students can see that the 2 in the hundreds place represents 200 dots, and they can develop a sense of the relative size of 200 dots.

Once students have a conceptual understanding of the number of ones contained inside each place, they move to drawings without dots. For example, the drawing below shows 176. Because these types of drawings do not need to be perfectly scaled, students can make them quickly. Grouping ten sticks and ones in subgroups of five helps to avoid errors and to make the drawings easier to read.

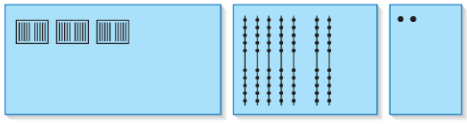
Students broaden their understanding of place value as they extend their models to include thousands. Students represent one thousand using a *thousand bar*. Students then apply these understandings to sketch models for numbers in the thousands. The model shows 2 thousands, 3 hundreds, 6 tens, and 8 ones, or 2,368.



from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

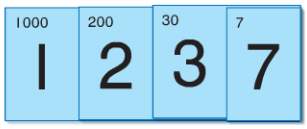
Base ten Units The power of the base ten system is in repeated bundling by ten: 10 tens make a unit called a hundred. Repeating this process of creating new units by bundling in groups of ten creates units called thousand, ten thousand, hundred thousand . . . .

Secret Code Cards Students explore place value by assembling Secret Code Cards to form multidigit numbers. The cards show place values. On the front of each card, the value of a number in a certain place appears, such as 300. The back of each Secret Code Card has a place value drawing representation of the number shown on the front. This drawing helps students to further understand the value of each number by showing a pictorial representation of the base ten form. To show 372, for example, students select cards representing 3 hundreds, 7 tens, and 2 ones. They can then show the number pictorially, in base ten form, or in standard form:



Hundreds Card Tens Card Ones Card

Each card has a small version of the number in the upper left corner. So even after the number 372 is assembled, students can see that the 3 represents 300, the 7 represents 70, and the 2 represents 2. Students continue to extend their understanding of place value as they use the cards to model numbers in the thousands.

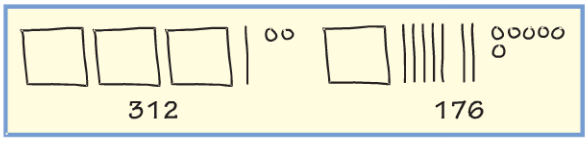


Using the cards is beneficial for students because the cards emphasize how the position of the digit in the number determines the value of the digit. For example, with the cards students can more easily see that a 2 on the hundreds card is 200, while a 2 on the tens card is 20.

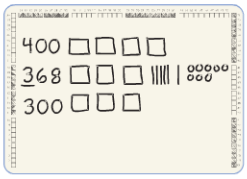
Comparing, Rounding, and Estimation

Lessons 1 5 6

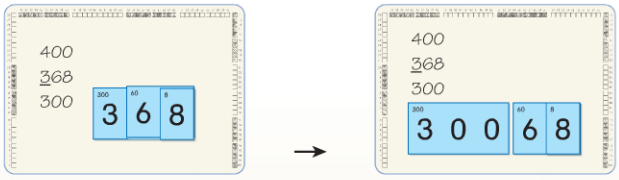
Comparing An understanding that, in the base ten system, one of a greater unit is always greater than nine or fewer of a lesser unit provides students with the foundation necessary to compare numbers of increasing value. To help students connect their understanding of place value to comparing, they are encouraged to make place value drawings. They can see from the drawings below that there are more hundreds in 312 than 176, so 312 > 176.



Rounding In this unit, students learn how to use rounding to estimate. They model rounding numbers using both place value drawings and Secret Code Cards. The model below helps students understand that since there are more than 5 sticks, 368 rounds up to 400.



In this example, students use Secret Code Cards to show that since 68 is closer to 100 than to 0, 368 rounds to 400.



Students use these models to extend their understanding of rounding to rounding 4-digit numbers to the nearest hundred and 2- and 3-digit numbers to the nearest ten.

Estimation Throughout this unit, students apply their understanding of rounding to estimate answers and to determine if an answer is reasonable. They learn that they can find an approximate answer to a problem involving addition or subtraction by rounding the numbers in the problem and either adding or subtracting. They can use that answer to estimate the solution or check a given solution to the problem.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

Comparing Comparing magnitudes of two-digit numbers draws on the understanding that 1 ten is greater than any amount of ones represented by a one-digit number. Comparing magnitudes of three-digit numbers draws on the understanding that 1 hundred (the smallest three-digit number) is greater than any amount of tens and ones represented by a two-digit number.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

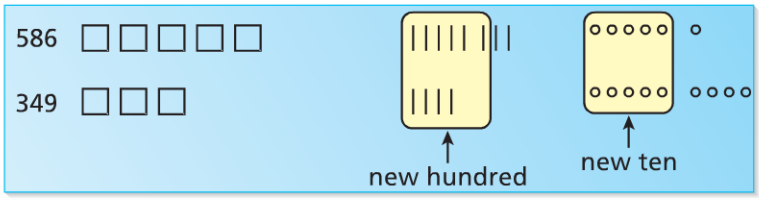
Rounding Students use their place value understanding to round numbers to the nearest 10 or 100. They need to understand that when moving to the right across the places in a number (e.g., 456), the digits represent smaller units. When rounding to the nearest 10 or 100, the goal is to approximate the number by the closest number with no ones or no tens and ones (e.g., so 456 to the nearest ten is 460; and to the nearest hundred is 500).

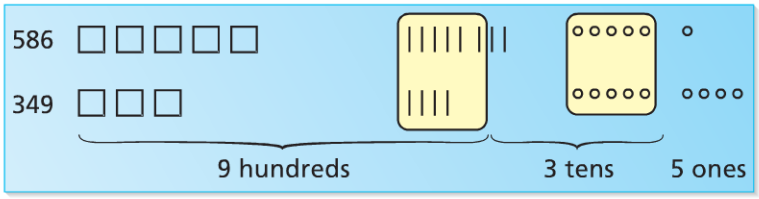
Addition of Whole Numbers

Lessons 7 8 9 10

Computations The uniformity of the base ten system facilitates understanding of place value concepts, but it also provides the foundation for successfully completing standard algorithms for computation within the base ten system. Once students understand that numbers are composed of ones, tens, hundreds, and so on, they can use this understanding to decompose and compose units in computations.

Modeling Addition Before numeric methods are presented, Lesson 7 encourages students to use place value drawings to add. The following place value drawing shows how students add 586 and 349. The drawing allows students to visualize the regrouping of 10 ones as 1 ten and 10 tens as 1 hundred.





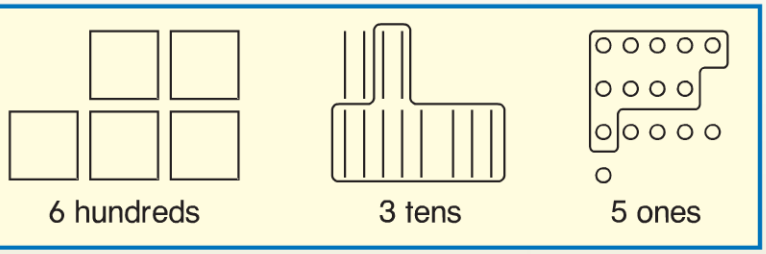
from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

Computations Standard algorithms for base ten computations with the four operations rely on decomposing numbers written in base ten notation into base ten units. The properties of operations then allow any multidigit computation to be reduced to a collection of single-digit computations. These single-digit computations sometimes require the composition or decomposition of a base ten unit.

Numeric Addition Methods After students explore addition using place value drawings, three methods for adding numbers are presented. The variety of algorithms is beneficial for students because it allows them to choose the algorithm that best suits their learning style and the one that feels the most natural to them. The algorithms themselves emphasize grouping and ungrouping to address common errors that are learning obstacles for students. The examples below show how to use these three numeric methods to add 249 and 386. Students learn that the order in which the steps of the method are recorded does not change the value of the answer.

Show All Totals Method	New Groups Below Method	New Groups Above Method
$\begin{array}{r} 249 \\ +\ 386 \\ \hline 500 \\ 120 \\ +\ 15 \\ \hline 635 \end{array}$	$\begin{array}{r} 249 \\ +\ 386 \\ \hline 635 \end{array}$	$\begin{array}{r} {\scriptstyle 1\ 1} \\ 249 \\ +\ 386 \\ \hline 635 \end{array}$
Students add in each place, record the total for each place, then add these totals to find the sum. This can be done from the left or right.	Students record a regrouped digit on the line below the next left column beginning from the right.	The regrouped digit is recorded above the next left column.

Proof Drawings Proof drawings are used to visually illustrate the grouping process in addition and the ungrouping process in subtraction. At first, students link each step of a proof drawing to each step of a numerical method. Students then begin to do only the numerical method, but they can think of a drawing to self-correct. Occasionally, it is helpful for students to make a proof drawing to explain their numerical method to someone else and to keep the meanings attached to the numerical method. Students use a proof drawing, such as the one below, to show that 249 + 386 = 635



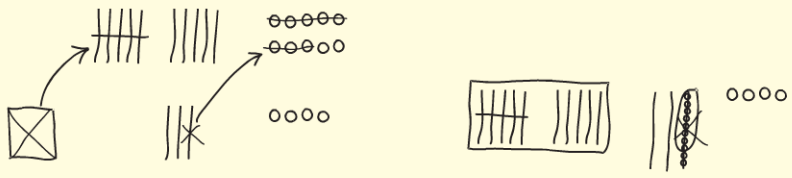
***from* THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN**

Algorithms Students continue adding and subtracting within 1000. They achieve fluency with strategies and algorithms that are based on place value, properties of operations, and/or the relationship between addition and subtraction.

Subtraction of Whole Numbers

Lessons 11 12 13 14

Modeling Subtraction As with addition, before numeric subtraction methods are presented, students are encouraged to use place value drawings to subtract. The following place value drawings show two ways that students might model 134 – 58. Notice that, although students' models look different, in each model, they ungroup a hundred and a ten and cross out to show taking away 58.



Student 1 Student 2

Numeric Subtraction Methods Lesson 11 presents numeric methods for subtracting numbers in the hundreds. The subtraction algorithms parallel the addition algorithms students have learned. This parallel presentation benefits students because they can use their understanding of place value and addition to subtract.

In numeric addition methods, students use composition to group units of lesser values to units of greater values, for example, grouping 10 ones as 1 ten, or 10 tens as 1 hundred. With subtraction, students use the opposite action. They decompose numbers to ungroup units of greater value into units of lesser value. For example, they ungroup 1 ten to 10 ones or 1 hundred to 10 tens. The following examples show how students use three numeric methods to subtract 58 from 134.

Expanded Method

Ungroup right to left

$$\begin{array}{r} 134 \\ -\,58 \end{array} = \begin{array}{r} {}^{0}\cancel{100} + {}^{120}\cancel{30} + {}^{14}\cancel{4} \\ -\quad 50 + 8 \end{array}$$

$$70 + 6 = 76$$

Ungroup left to right

$$\begin{array}{r} 134 \\ -\,58 \end{array} = \begin{array}{r} {}^{0}\cancel{100} + {}^{120}\cancel{\overset{130}{30}} + {}^{14}\cancel{4} \\ -\quad 50 + 8 \end{array}$$

$$70 + 6 = 76$$

Students write each number in expanded form. They ungroup as needed to subtract. They subtract in each place. Then they add the differences.

Ungroup First Method

Ungroup left to right

$$\begin{array}{r} 134 \\ -\,58 \end{array} \rightarrow \begin{array}{r} {}^{0\,13}\cancel{134} \\ -\,58 \end{array} \rightarrow \begin{array}{r} {}^{0\,13\,14}\cancel{134} \\ -\,58 \\ \hline 76 \end{array}$$

Ungroup right to left

$$\begin{array}{r} 134 \\ -\,58 \end{array} \rightarrow \begin{array}{r} {}^{2\,14}\cancel{134} \\ -\,58 \end{array} \rightarrow \begin{array}{r} {}^{0\,12\,14}\cancel{134} \\ -\,58 \\ \hline 76 \end{array}$$

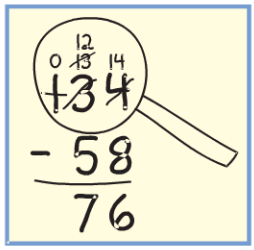
Students determine the necessary ungrouping. Then they subtract in any direction.

Common U.S. Method

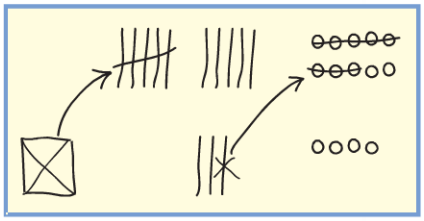
$$\begin{array}{r} 134 \\ -\,58 \end{array} \rightarrow \begin{array}{r} {}^{2}\cancel{134} \\ -\,58 \\ \hline 6 \end{array} \rightarrow \begin{array}{r} {}^{0\,12}\cancel{134} \\ -\,58 \\ \hline 76 \end{array}$$

Students ungroup tens, subtract ones, ungroup hundreds, subtract tens, and so on.

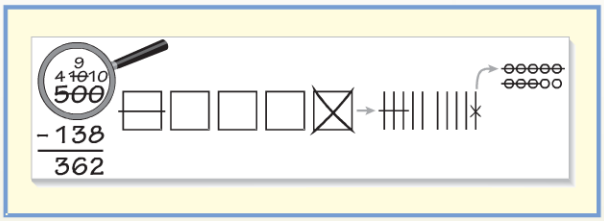
Common Errors To address students' common errors in problems where it is necessary to ungroup multiple times, students are encouraged to do all the ungrouping before subtracting. Students can ungroup either from left to right or from right to left. Once everything is ungrouped, students can subtract the places in any order. To facilitate students' decisions about when to ungroup, students draw a "magnifying glass" around the top number. This enables them to focus in on the digits in the first number to determine if there are enough in each place value to subtract.



As with addition, students are asked to make proof drawings to show ungrouping and to verify that their answers are correct.



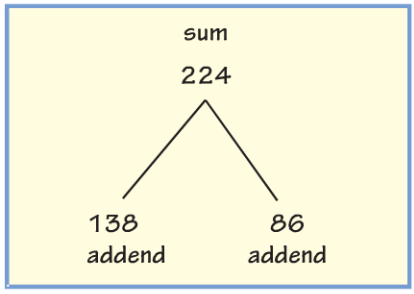
Subtracting Across Zeros In Lesson 12, students subtract across zeros. The work that students have done connecting numeric methods to conceptual understanding facilitates understanding of how to subtract across zeros. The following example shows how students can use the Ungroup First method to subtract 138 from 500. Asking students to use a proof drawing along with the numeric method encourages them to connect the conceptual underpinnings of subtraction with each algorithmic step.



Relating Addition and Subtraction

Lessons 14 16

Math Mountains In Lesson 14, a Math Mountain is introduced to help students conceptualize the relationship between addition and subtraction. Students visualize the total at the top of the mountain breaking into two pieces, one of which rolls down one side and one of which rolls down the other side. The following shows a Math Mountain for 224, 138, and 86.

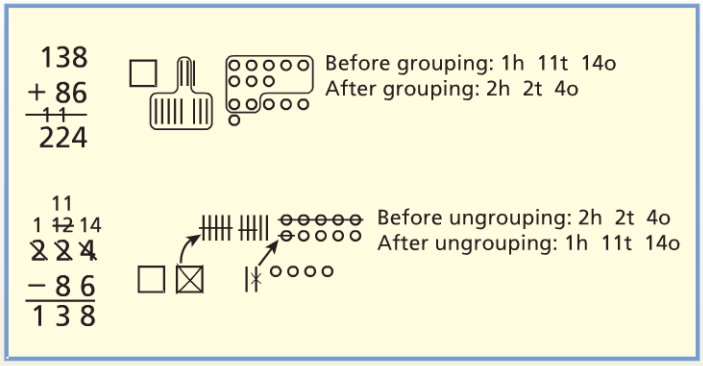


Students can use the Math Mountain to write related addition and subtraction equations, such as:

138	+	86	=	224		224	−	86	=	138	
addend		addend		sum	and	sum		addend		addend	

By thinking about addition and subtraction in terms of a total and two parts, students can understand the relationship between addition and subtraction as well as use the Math Mountain to represent the relationships in word problems.

Grouping and Ungrouping Students analyze the proof drawings for related addition and subtraction problems to connect the grouping in addition with the ungrouping in subtraction. As they develop this skill, they begin to see that the before grouping model for addition correlates to the after ungrouping model for subtraction. The after grouping model for addition correlates to the before ungrouping model for subtraction. Notice that when adding in the example below, students group 1 hundred, 11 tens, and 14 ones to get 2 hundreds, 2 tens, and 4 ones. When subtracting, students ungroup 2 hundreds, 2 tens, and 4 ones as 1 hundred, 11 tens, and 14 ones.



Problem Solving

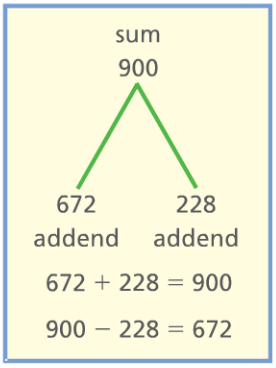
Lessons 5 6 9 10 15 16 17

Problem Solving Plan In *Math Expressions* a research-based problem solving approach that focuses on problem types is used.

- Interpret the problem
- Represent the situation
- Solve the problem
- Check that the answer makes sense

Choosing the Operation The unit presents, in the same lesson, word problems that can be solved using addition and subtraction. This gives students the opportunity to decide which operation to use. Students learn to attend closely to the structure of the problem and use a model to help them to determine what operation is indicated. Students are asked to show their work by writing an equation to represent the problem.

Multistep Problems The equations that students write for problems with more than one step may include more than one operation. Sometimes the order in which these operations are completed is not important. Students are encouraged to consider a variety of ways to solve different problems.



Focus on Mathematical Practices

Lesson 18

The standards for Mathematical Practice are included in every lesson of this unit. However, there is an additional lesson that focuses on all eight Mathematical Practices. In this lesson, students use what they know about adding and subtracting whole numbers to solve problems involving maps and email.