## **Getting Ready to Teach Unit 2**

#### **Learning Path in the Common Core Standards**

In this unit, students continue to participate in testing and goal direction practice in school and at home. A variety of practice sheets, check sheets and routines help students learn the basic multiplications and divisions and help students keep track of their progress. This is also an important opportunity for students to become self-directed and organized.

Students also continue practicing with the factors covered in Unit 1, learn multiplications and divisions for 6, 7, and 8, and extend their skills with an introduction of multiplying a one-digit number by a multiple of 10.

Visual models and real world situations are used throughout the unit to illustrate important multiplication and division concepts.

#### **Help Students Avoid Common Errors**

Math Expressions gives students opportunities to analyze and correct errors, explaining why the reasoning was flawed.

In this unit we use Puzzled Penguin to show typical errors that students make. Students enjoy teaching Puzzled Penguin the correct way, why this way is correct and why Puzzled Penguin made the error. Common errors are presented in Puzzled Penguin features in the following lessons:

- ▶ **Lesson 4:** Confused + with × sign and added instead of multiplying
- **Lesson 10:** Did not follow the rules for the Order of Operations and added before multiplying when solving a two step problem

In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As a part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.

#### **Math Expressions VOCABULARY**

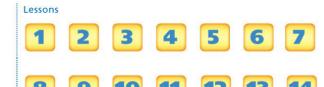
As you teach this unit, check understanding of these terms:

- array problem
- equal groups problem
- area problem

See the Teacher Glossary.



# Basic Multiplications and Divisions 6s, 7s, and 8s



Path to Fluency The hardest facts, 6s, 7s, and 8s, are introduced and practiced in Unit 2. The basic facts 5s, 2s, 9s, 10s, 3s, 4s, 1s, and 0s were introduced and practiced from easiest to hardest in Unit 1.

When first learning the facts, students use count-bys with diagrams to give the multiplication meaning and to connect symbols with words and equations. Next they look for patterns, rules, or strategies to make learning that fact easier. Students practice and check fluency of the facts using a routine with a variety of tools such as Study Sheets, Check Sheets and Strategy Cards.

Students should work with one fact at a time: studying, practicing, and checking recall. Then students should combine checking previously learned facts with new facts. Multiplication and division are taught together to make the process meaningful and faster. Each division is just finding an unknown factor. Some students may not learn all the basic multiplications and divisions this year.

Unit 2 has Diagnostic Checkups for Multiplication and Division to check fluency on all basic facts in Lesson 14.

#### from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Fluency Fluency in Grade 3 involves a mixture of just knowing some answers from knowing other answers, knowing some answers from patterns (e.g., "multiplying 1 yields the same number"), and knowing some answers from the use of strategies.

#### Practice Materials for Learning the Basic Multiplications and Divisions

Lessons

1 2 3 4 5 6 7

8 9 10 11 12

Study Plans Each day students will fill out a study plan at the top of a homework page, indicating which basic multiplications and divisions he or she will study that evening at home. At first it contains just the count-bys, multiplications, and divisions for the new number introduced. Later it will be the new number and any count-bys, multiplications, or divisions they do not recall when tested by their partner during the Check Up.

When a student has finished practicing/studying, the Homework Helper should sign the study plan.

1–1 Name	Date
Homework	
Study Plan 5s count-bys	
5s multiplications	
	Homework Helper

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Practice Charts In Lessons 1, 3, and 5, each time a new number is introduced, a student's Homework page will include a practice chart. See Teacher Edition page 8 for an explanation of how to practice the count-bys, multiplications, and divisions by covering the answers with a pencil and sliding it.

	In Order	Mixed Up
	1 × 6 = 6	9 × 5 = 54
	2 × 6 = 12	5 × 6 = 30
	3 × 6 = 18	2 × 6 = 12
	4 × 6 = 24	$7 \times 6 = 42$
<b>6</b> s	5 × 6 = 30	$4 \times 6 = 24$
	6 × 6 = 36	6 × 6 = 36
	7 × 6 = 42	$10 \times 6 = 60$
	8 × 6 = 48	8 × 6 = 48
	9 × 6 = 54	1 × 6 = 6
	10 × 6 = 60	3 × 6 = 18

Study Sheets Students use both a class and home study sheet, which includes 3 or 4 practice charts on one page. This sheet can be used to practice all the count-bys, multiplications, and divisions or to practice just the ones a student doesn't know.

5s			25			
Count-bys	Mixed Up ×	Mixed Up ÷	Count-bys	Mixed Up ×	Mixed Up ÷	
$1 \times 5 = 5$	2 × 5 = 10	10 ÷ 5 = 2	1 × 2 = 2	7 × 2 = 14	20 ÷ 2 = 10	
2 × 5 = 10	9 x 5 = 45	35 + 5 = 7	$2 \times 2 = 4$	1 × 2 = 2	2+2 = 1	
3 × 5 = 15	$1 \times 5 = 5$	50 + 5 = 10	$3 \times 2 = 6$	$3 \times 2 = 6$	6 + 2 = 3	
$4 \times 5 = 20$	$5 \times 5 = 25$	5 + 5 = 1	$4 \times 2 = 8$	5 x 2 = 10	16 + 2 = 8	
$5 \times 5 = 25$	$7 \times 5 = 35$	20 + 5 = 4	5 x 2 = 10	6 × 2 = 12	12 + 2 = 6	
$6 \times 5 = 30$	3 x 5 = 15	15 ÷ 5 = 3	6 x 2 = 12	8 x 2 = 16	$4 \div 2 = 2$	
$7 \times 5 = 35$	10 x 5 = 50	30 ÷ 5 = 6	7 × 2 = 14	$2 \times 2 = 4$	10 ÷ 2 = 5	
$8 \times 5 = 40$	6 x 5 = 30	40 ÷ 5 = 8	8 x 2 = 16	10 × 2 = 20	$8 \div 2 = 4$	
$9 \times 5 = 45$	$4 \times 5 = 20$	25 ÷ 5 = 5	9 x 2 = 18	4 x 2 = 8	$14 \div 2 = 7$	
0 x 5 = 50	$8 \times 5 = 40$	45 + 5 = 9	10 × 2 = 20	9 × 2 = 18	18 + 2 = 9	

OIII	iework H	ome Signature Sheet	
	Count-Bys Homework Helper	Multiplications Homework Helper	Divisions Homework Helper
$\top$			

A routine is built into this program so each day at school and at home students practice count-bys, multiplications, and divisions and are tested when ready. When a student is ready for a Check Up on a number, a student's partner or Homework Helper tests the student, marking any missed exercises lightly with a pencil. If a student gets all the answers in a column correct, the partner or Homework Helper signs the Signature Sheet or the Home Signature Sheet.

#### from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

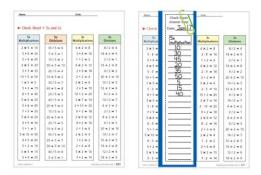
**Building Fluency** Fluency may be reached by becoming fluent for each number (e.g. the 2s, the 5s, etc.) and then extending the fluency to several, then all numbers mixed together. Organizing practice so that it focuses most heavily on understood but not yet fluent products and unknown factors can speed learning.

Signature Sheet When a student gets all the answers in a column on the Study Sheet correct, the partner or Homework Helper signs the appropriate column on the Signature Sheet. When signatures are in all the columns, the student turns the Signature Sheet into the teacher so there is a record that the multiplications and divisions have been mastered.

#### **Signature Sheet**

	Count-Bys Partner	Multiplications Partner	Divisions Partner	Multiplications Check Sheets	Divisions Check Sheets
5s				1:	1:
2s				1:	1:
10s				2:	2:
9s				2:	2:

Check Sheet In Lessons 3, 6, and 7, when a student has signatures for a fact, they use an answer strip to complete a Check Sheet for that fact and place it in their Fluency Progress Folder. Both the class and home check sheets include columns of 20 multiplications and divisions in mixed order.

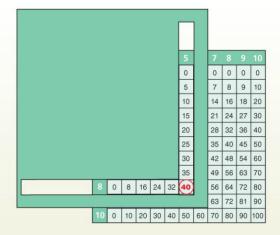


Targets Students use the Targets with a Multiplication Table to practice multiplications and divisions and to see inverse and commutative relationships. By covering the Target circle, students can check on whether they know the product for two factors. By covering one end of the Target, they can check on a related division.

Students can take one Target home and use it with the Multiplication Table on the inside back cover of the Homework and Remembering book or with a copy of TRB M12. They can use the Targets throughout the year to maintain fluency with basic multiplications and divisions.

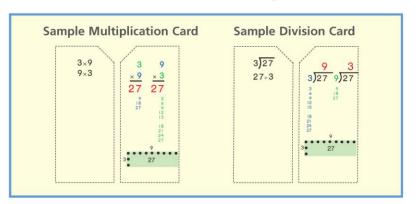
from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

The Relationship Between
Multiplication and Division The
extensive work relating
multiplication and division
means that division can be
solved by thinking of the related
multiplication. Multiplication and
division can be learned at the same
time and can reinforce each other.



Strategy Cards In Unit 1, students are introduced to the Strategy Cards. Students can use the cards to practice multiplications and divisions. They should sort their cards into three piles: those with answers they know quickly, those with answers they know slowly, and those with answers they don't know yet. As they practice, they may be able to move some of the cards into other piles.

There is a home set of Strategy Cards on Homework and Remembering pages 41–66. As part of their homework, students should cut out the cards and use them to study.



#### from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Studying Facts Facts should not be instilled divorced from their meanings, but rather the outcome of a carefully designed learning process that heavily involves the interplay of practice and reasoning.

Dashes When students have successfully completed all Check Sheets, they complete a Dash and place it in their Fluency Progress folder. The Dashes are 20 multiplications or divisions in mixed order. Dashes with the same facts in a different order are included so students can time themselves and try to improve their speed on the next dash with the same facts. For example, 12, 12A, 12B, and 12C all have the same facts, but are in a different order to test fluency and time for recall.

#### from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Checking Fluency It is important to push sensitively and encouragingly toward fluency of the designated numbers in each lesson, recognizing that fluency will be a mixture of kinds of thinking which may differ across students.

### PATH to Dashes 9A-12A

Complete each Dash. Check your answers on page 82.

	Dash 9A 2s, 5s, 9s, 10s Multiplications					
a.	9	×	9	=	_	
b.	4	*	5	=		
c.	10	•	3	=		
d.	3	×	9	=		

Dash 10A 2s, 5s, 9s, 10s Divisions					
a.	30	/	5	=	
b.	18	÷	2	=	
c.	40	/	5	=	
d.	6	÷	2	=	

0s, 1s, 3s, 4s Multiplications					
a.	0	×	4	=	
b.	4	*	9	=	
c.	3	•	8	=	_
d.	3	×	0	=	

Dash 11A

Dash 12A 1s, 3s, 4s Divisions						
a. 10 /	1	=				
b. 40 ÷	4	=				
c. 12 /	3	=	_			
d. 6 ÷	3	=				

## PATH to Fluency Dashes 9B-12B

Complete each multiplication and division Dash. Check your answers on page 126.

Dash 9B 2s, 5s, 9s, 10s Multiplications	Dash 10B 2s, 5s, 9s, 10s Divisions	Dash 11B 0s, 1s, 3s, 4s Multiplications	Dash 12B 1s, 3s, 4s Divisions
a. 6 × 2 =	a. 18 / 2 =	a. 7 × 1 =	a. 2 / 1 =
b. 9 • 4 =	b. 25 ÷ 5 =	b. 0 • 6 =	b. 28 ÷ 4 =
c. 8 * 5 =	c. 70 / 10 =	c. 4 * 4 =	c. 3 / 3 =
d. 1 × 10 =	d. 54 ÷ 9 =	d. 7 × 3 =	d. 1 ÷ 1 =
e. 2 • 7 =	e. 50 / 5 =	e. 3 • 1 =	e. 40 / 4 =

# Fluently Multiply and Divide within 100

Lessons 8



**Fluency Lessons** These lessons are included to reinforce facts learned, to give more time to students who need it to study and practice a fact, and to review strategies.

Independent activities are also included in these lessons.

**Independent Activities** Students who do not need extra teaching or practice for fluency after completing a Check Sheet may choose from one of the activity options shown below.

#### Go Ahead

Student Pairs go ahead to the next fact, using the same plan as for 2s and 5s: study, partner check, complete Check Sheet for that fact.

#### Go for Speed

**Student Pairs** go for speed. They can use an answer strip from the back of the Student Activity Book or Activity Workbook, or Check Sheet Answer Strips (TRB M7) to complete a Check Sheet for one of the facts and record the time it takes to complete it. Then they can complete it again using another answer strip and compare it with the first time.

#### Invent a Game or Play a Game

**Student Pairs** can invent a game to learn or practice multiplications and divisions. Students can also play the games *Solve the Stack* and *High Card Wins* introduced in Unit 1, Lesson 13.

#### Write a Word Problem

**Student Pairs** can write word problems that can be solved using a multiplication or division they know. Then exchange to solve.

#### Invent Rhymes or Songs

**Student Pairs** write rhymes or songs that will help everyone remember the hardest multiplications and division facts and lead the class to practice them.

#### Create Poster

**Student Pairs** can make pictographs or function tables, where each picture represents 3, 4, 6, 7, or 8, and lead the class to practice with them.

#### Coach

A student who has achieved fluency can function as a coach to help less advanced students practice and plan effective practice methods. They can help students use different strategies. This individualized help and encouragement can be very helpful to a student in need of both.

#### from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Patterns and Strategies Mastering this material, and reaching fluency in single digit multiplications and related divisions with understanding, may be quite time consuming because there are no general strategies for multiplying or dividing all single-digit numbers as there are for addition and subtraction. Instead, there are many patterns and strategies dependent upon specific numbers. So it is imperative that extra time and support be provided if needed.

#### Multiplication Strategies for 6s, 7s, and 8s

Lessons









#### Starting with a Known Count-by and Then Counting On

When students are give a multiplication example they don't know, we encourage them to try to figure out the answer using the count-bys they already know.

$$6 \times 6 = \square$$

Start with a 5s count-by you know:  $5 \times 6 = 30$ .

Then count by 6 from there: 30 plus 6 more is 36.

So, 
$$6 \times 6 = 36$$
.

#### **Using a Known Multiplication and Doubling**

Another strategy we encourage students to use when they see a new multiplication they do not know is to see if using doubles can help. For example, for a 6s multiplication they don't know, doubling a 3s multiplication can give the correct answer.

$$6 \times 6 = \square$$
  
 $6 \times 6$  is twice  $6 \times 3$ .  
 $6 \times 3 = 18$ . Then  $18 + 18 = 36$ . So,  $6 \times 6 = 36$ .

#### **Combining Two Known Multiplications**

A strategy to find the answer to a new multiplication or a multiplication they don't recall is for students to think about two multiplications they do know. The products can be added to find the product of the example they don't know or recall.

$$6 \times 6 = \square$$

$$4 \times 6 = 24$$

$$2 \times 6 = 12$$

$$6 \times 6 = 36$$

$$4 \text{ sixes are } 24$$

$$2 \text{ sixes are } 12$$

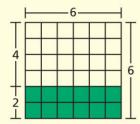
$$6 \text{ sixes are } 36$$

#### from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Distributive Property Area problems where regions are partitioned by unit squares are foundational for Grade 3 standards because area is used as a model for single-digit multiplication and division strategies, such as decomposing to find the sum of two known facts.

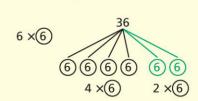
#### **Using a Drawing to Combine Two Known Multiplications**

Here are two ways to show combining two known multiplications to find the product of a multiplication they do not know.



unshaded area:  $4 \times 6 = 24$ shaded area:  $2 \times 6 = 12$ 

 $6 \times 6 = 36$ 



#### **Explanation:**

6 groups of 6 is 4 groups of 6 plus 2 groups of 6.

#### Starting with a Known Count-by and Adding On or Subtracting From

Add 6 to the 6s multiplication before or subtract from the multiplication ahead.

 $5 \times 6 = 30$ , add 6 to get 36.

So,  $6 \times 6 = 36$ .

total area:

#### **Patterns**









#### **Using Patterns**

Our goal is to encourage students to use patterns, such as countbys, doubling, and patterns in the multiplications and divisions they already know to make proficiency with these facts easier. Using patterns can be a helpful strategy that they may use whenever they encounter examples or concepts that are new or difficult.

#### from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Patterns and Strategies Mastering this material, and reaching fluency in single digit multiplications and related divisions with understanding, may be quite time consuming because there are no general strategies for multiplying or dividing all single-digit numbers as there are for addition and subtraction. Instead, there are many patterns and strategies dependent upon specific numbers. So it is imperative that extra time and support be provided if needed. itelits Flai

# Strategies for Multiplying by a Multiple of Ten



#### **Using Place Value and Properties**

When we multiply with multiples of 10, we can use place value to rewrite the factors. Then we can use the Associative and Commutative Properties to rearrange the factors so that we can multiply the product of a basic fact by 10.

$$2 \times 3 = (2 \times 1) \times (3 \times 1) = (2 \times 3) \times (1 \times 1) = 6 \times 1 = 6$$
  
 $2 \times 30 = (2 \times 1) \times (3 \times 10) = (2 \times 3) \times (1 \times 10) = 6 \times 10 = 60$ 

#### **Using a Shortcut**

Find the basic multiplication product. Then multiply by 10.

$$6 \times 10 = 60$$

#### Represent Word Problems Involving Multiplication and Division

Lessons







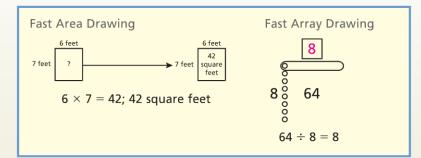




In *Math Expressions* a research-based problem solving approach that focuses on problem types is used.

- Interpret the problem
- Represent the situation
- Solve the problem
- Check that the answer makes sense

Students using *Math Expressions* are taught a variety of ways to represent word problems. Some are conceptual in nature (making math drawings), while others are symbolic (writing equations). Students move from using math drawings to solving problems symbolically with equations. The following are math drawings students use to represent multiplication and division word problems in this unit.



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#### Represent and Solve Two Step Word **Problems with Four Operations**

Lessons



Students solve two step problems by writing two separate expressions or by writing one expression with two operations.

Students begin by writing first step questions.

Write the first step question and answer. Then solve the problem.

A roller coaster has 7 cars. Each car has 4 seats. If there were 3 empty seats, how many people were on the roller coaster?

How many seats are on the roller coaster? 28

#### Focus on **Mathematical Practices**





The Standards for Mathematical Practice are included in every lesson of this unit. However, there is an additional lesson that focuses on all eight Mathematical Practices. In this lesson, students describe strategies for multiplying and dividing by 6, 7, and 8 and complete a chart using information provided in a pictogragh.

#### from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Variables Students in Grade 3 begin the step to formal algebraic language by using a letter for the unknown quantity in expressions or equations for one and two step problems, and use the Order of Operations. The symbols of arithmetic, × or • for multiplication and ÷ or / for division are used.