

Getting Ready to Teach Unit 1

Learning Path in the Common Core Standards

In this unit and Unit 2, students participate in testing and goal direction practice in school and at home. A variety of practice sheets, check sheets and routines help students learn the basic multiplications and divisions and help students keep track of their progress. This is also an important opportunity for students to become self-directed and organized.

Students also learn how to use different strategies for multiplying and dividing, how multiplication and division are related, and how to use math drawings and equations to represent and solve word problems.

Visual models and real world situations are used throughout the unit to illustrate important multiplication and division concepts.

Help Students Avoid Common Errors

Math Expressions gives students opportunities to analyze and correct errors, explaining why the reasoning was flawed.

In this unit we use Puzzled Penguin to show typical errors that students make. Students enjoy teaching Puzzled Penguin the correct way, why this way is correct and why Puzzled Penguin made the error. Common errors are presented in Puzzled Penguin features in the following lessons:

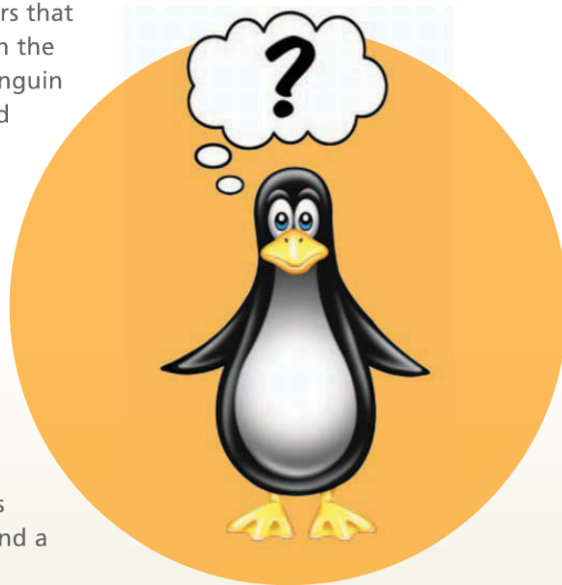
- ▶ **Lesson 4:** Uses an incorrect factor in the related multiplication to solve a division
- ▶ **Lesson 7:** Writes a word problem that cannot be solved using $40 \div 10$
- ▶ **Lesson 12:** Combines multiplication equations by adding both factors

In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As a part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.

Math Expressions VOCABULARY

- Equal Shares drawing
- count-bys
- Fast Array drawing
- Equal Groups drawing

See the Teacher Glossary.



Lessons



Basic Multiplications and Divisions

Path to Fluency The basic facts in Unit 1 are introduced and practiced from easiest to hardest: 5s, 2s, 9s, 10s, 3s, 4s, 1s, and 0s. The harder facts 6s, 7s, and 8s are introduced and practiced in Unit 2.

When first learning the facts, students use count bys with diagrams to give the multiplication meaning and to connect symbols with words and equations. Next they look for patterns, rules, or strategies to make learning that fact easier. Students practice and check fluency of the facts using a routine with a variety of tools such as Study Sheets, Check Sheets and Strategy Cards.

Students should work with 1 fact at a time: studying, practicing, and checking recall. Then students should combine checking previously learned facts with new facts. Multiplication and division are taught together to make the process meaningful and faster. Each division is just finding an unknown factor. Some students may not learn all the basic multiplications and divisions this year.

from the Progression S For the Common Core State Standard S on oPeration S and algebrai C t hinking

the meaning of multiplication

Students focus on understanding the meaning and properties of multiplication and division and on finding products of single-digit multiplying and related quotients. These skills and understandings are crucial; students will rely on them for years to come as they learn to multiply and divide with multidigit whole numbers and to add, subtract, multiply and divide with fractions and with decimals in later grades.

Lessons



Practice Materials and Routines for Learning the Basic Multiplications and Divisions

Study Plans Each day students will fill out a study plan at the top of a homework page, indicating which basic multiplications and divisions he or she will study that evening at home. At first it contains just the count-bys, multiplications, and divisions for the new number introduced. Later it will be the new number and any count-bys, multiplications, or divisions they do not recall when tested by their partner during the Check Up.

When a student has finished practicing/studying, the Homework Helper should sign the study plan.

1-1

Name _____

Date _____

Homework

Study Plan

5s count bys

5s multiplications

Homework Helper _____

Practice Charts In Lessons 1, 5, 7, 8, 10, 12, and 15, each time a new number is introduced, a student's Homework page will include a practice chart. See Teacher Edition page 8 for an explanation of how to practice the count-bys, multiplications, and divisions by covering the answers with a pencil and sliding it.

	In Order	Mixed Up
5s	$1 \times 5 = 5$	$9 \times 5 = 45$
	$2 \times 5 = 10$	$5 \times 5 = 25$
	$3 \times 5 = 15$	$2 \times 5 = 10$
	$4 \times 5 = 20$	$7 \times 5 = 35$
	$5 \times 5 = 25$	$4 \times 5 = 20$
	$6 \times 5 = 30$	$6 \times 5 = 30$
	$7 \times 5 = 35$	$10 \times 5 = 50$
	$8 \times 5 = 40$	$8 \times 5 = 40$
	$9 \times 5 = 45$	$1 \times 5 = 5$
	$10 \times 5 = 50$	$3 \times 5 = 15$

from the PROGRESS for the COMMON CORE State Standards on Operations and Algebraic Thinking

Building fluency Fluency may be reached by becoming fluent for each number (e.g. the 2s, the 5s, etc) and then extending the fluency to several, then all numbers mixed together. Organizing practice so that it focuses most heavily on understood but not yet fluent products and unknown factors can speed learning.

Study Sheets Students use both a class and home study sheet, which includes 3 or 4 practice charts on one page. This sheet can be used to practice all the count-bys, multiplications, and divisions or to practice just the ones a student doesn't know.

Home Study Sheet A								
5s			2s					
Count-bys	Mixed Up \times	Mixed Up \div	Count-bys	Mixed Up \times	Mixed Up \div			
$1 \times 5 = 5$	$2 \times 5 = 10$	$10 \div 5 = 2$	$1 \times 2 = 2$	$7 \times 2 = 14$	$20 \div 2 = 10$			
$2 \times 5 = 10$	$9 \times 5 = 45$	$35 \div 5 = 7$	$2 \times 2 = 4$	$1 \times 2 = 2$	$2 \div 2 = 1$			
$3 \times 5 = 15$	$1 \times 5 = 5$	$50 \div 5 = 10$	$3 \times 2 = 6$	$3 \times 2 = 6$	$6 \div 2 = 3$			
$4 \times 5 = 20$	$5 \times 5 = 25$	$5 \div 5 = 1$	$4 \times 2 = 8$	$5 \times 2 = 10$	$16 \div 2 = 8$			
$5 \times 5 = 25$	$7 \times 5 = 35$	$20 \div 5 = 4$	$5 \times 2 = 10$	$6 \times 2 = 12$	$12 \div 2 = 6$			
$6 \times 5 = 30$	$3 \times 5 = 15$	$15 \div 5 = 3$	$6 \times 2 = 12$	$8 \times 2 = 16$	$4 \div 2 = 2$			
$7 \times 5 = 35$	$10 \times 5 = 50$	$30 \div 5 = 6$	$7 \times 2 = 14$	$2 \times 2 = 4$	$10 \div 2 = 5$			
$8 \times 5 = 40$	$6 \times 5 = 30$	$40 \div 5 = 8$	$8 \times 2 = 16$	$10 \times 2 = 20$	$8 \div 2 = 4$			
$9 \times 5 = 45$	$4 \times 5 = 20$	$25 \div 5 = 5$	$9 \times 2 = 18$	$4 \times 2 = 8$	$14 \div 2 = 7$			
$10 \times 5 = 50$	$8 \times 5 = 40$	$45 \div 5 = 9$	$10 \times 2 = 20$	$9 \times 2 = 18$	$18 \div 2 = 9$			

1-3	Name _____	Date _____	
Homework			
Home Signature Sheet			
	Count-Bys Homework Helper	Multiplications Homework Helper	Divisions Homework Helper
0			
1			

A routine is built into this program so each day at school and at home students practice count-bys, multiplications, and divisions and are tested when ready. When a student is ready for a Check Up on a number, a student's partner or Homework Helper tests the student marking any missed exercises lightly with a pencil. If a student gets all the answers in a column correct, the partner or Homework Helper signs the Signature Sheet or the Home Signature Sheet.

Signature Sheet When a student gets all the answers in a column on the Study Sheet correct, the partner or Homework Helper signs the appropriate column on the Signature Sheet. When signatures are in all the columns, the student turns the Signature Sheet in to the teacher so there is a record that the multiplications and divisions have been mastered.

Signature Sheet

	Count-Bys Partner	Multiplications Partner	Divisions Partner	Multiplications Check Sheets	Divisions Check Sheets
5s				1:	1:
2s				1:	1:
10s				2:	2:
9s				2:	2:

Check Sheet In Lessons 5, 8, 9, 12, and 17, when a student has signatures for a fact they use an answer strip to complete a Check Sheet for that fact and place it in their Fluency Progress Folder. Both the class and home check sheets include columns of 20 multiplications and divisions in mixed order.

from THE progre SSion 5 for
The Common Core STaTe
Standard 5 on opera Tion 5
and algebrai C THinking

The relationship between multiplication and division

The extensive work relating multiplication and division means that division can be solved by thinking of the related multiplication. Multiplication and division can be learned at the same time and can reinforce each other.

Targets In Lesson 6, students begin using their Targets with a Multiplication Table to practice multiplications and divisions they have studied so far and to see inverse and commutative relationships. By covering the Target circle, students can check on whether they know the product for two factors. By covering one end of the Target, they can check on a related division.

Students can take one Target home and use it with the Multiplication Table on the inside back cover of the Homework and Remembering book or with a copy of TRB M12. They can use the Targets with a multiplication table or scrambled multiplication table throughout the year to maintain fluency with basic multiplications and divisions.

Strategy Cards In Lesson 11, students are introduced to the Strategy Cards. Students can use the cards to practice multiplications and divisions. They should sort their cards into three piles: those with answers they know quickly, those with answers they know slowly, and those with answers they don't know yet. As they practice, they may be able to move some of the cards into other piles.

There is a home set of Strategy Cards on Homework and Remembering pages 41–66. As part of their homework, students should cut out the cards and use them to study.

from the PROGRESS for the COMMON CORE State STANDARDS on OPERATIONS and ALGEBRA thinking

Studying facts Facts should not be instilled divorced from their meanings, but rather the outcome of a carefully designed learning process that heavily involves the interplay of practice and reasoning.

Sample Multiplication Card

$$\begin{array}{r} 3 \times 9 \\ 9 \times 3 \end{array}$$

$$\begin{array}{r} 3 \times 9 \\ 27 \end{array}$$

$$\begin{array}{r} 9 \times 3 \\ 27 \end{array}$$

Sample Division Card

$$\begin{array}{r} 3 \overline{)27} \\ 27 \end{array}$$

$$\begin{array}{r} 9 \overline{)27} \\ 27 \end{array}$$

Dashes When students have successfully completed all Check Sheets, they complete a Dash and place it in their Fluency Progress folder. The Dashes are 20 multiplications or divisions in mixed order. Dashes with the same facts in a different order are included so students can time themselves and try to improve their speed on the next Dash with the same facts. For example, 9A, 9B, 9C, and 9D have the same facts but are in a different order. A dash record sheet is included for students to record their speed and accuracy.

from the progressions for the Common Core standards on operations and algebraic thinking

Checking Fluency Fluency in Grade 3 involves a mixture of just knowing some answers from knowing other answers, knowing some answers from pattern (e.g., "multiplying 1 yields the same number"), and knowing some answers from the use of strategies. It is important to push sensitively and encouragingly toward fluency of the designated numbers in each lesson, recognizing that fluency will be a mixture of these kinds of thinking which may differ across students.

Dash 9A
2s, 5s, 9s, 10s
Multiplications

- a. $9 \times 9 =$ _____
b. $4 \times 5 =$ _____
c. $10 \times 3 =$ _____
d. $3 \times 9 =$ _____

Dash 10A
2s, 5s, 9s, 10s
Divisions

- a. $30 \div 5 =$ _____
b. $18 \div 2 =$ _____
c. $40 \div 5 =$ _____
d. $6 \div 2 =$ _____

Dash 11A
0s, 1s, 3s, 4s
Multiplications

- a. $0 \times 4 =$ _____
b. $4 \times 9 =$ _____
c. $3 \times 8 =$ _____
d. $3 \times 0 =$ _____

Dash 12A
1s, 3s, 4s
Divisions

- a. $10 \div 1 =$ _____
b. $40 \div 4 =$ _____
c. $12 \div 3 =$ _____
d. $6 \div 3 =$ _____

Dash 9B
2s, 5s, 9s, 10s
Multiplications

- a. $6 \times 2 =$ _____
b. $9 \times 4 =$ _____
c. $8 \times 5 =$ _____
d. $1 \times 10 =$ _____
e. $2 \times 7 =$ _____

Dash 10B
2s, 5s, 9s, 10s
Divisions

- a. $18 \div 2 =$ _____
b. $25 \div 5 =$ _____
c. $70 \div 10 =$ _____
d. $54 \div 9 =$ _____
e. $50 \div 5 =$ _____

Dash 11B
0s, 1s, 3s, 4s
Multiplications

- a. $7 \times 1 =$ _____
b. $0 \times 6 =$ _____
c. $4 \times 4 =$ _____
d. $7 \times 3 =$ _____
e. $3 \times 1 =$ _____

Dash 12B
1s, 3s, 4s
Divisions

- a. $2 \div 1 =$ _____
b. $28 \div 4 =$ _____
c. $3 \div 3 =$ _____
d. $1 \div 1 =$ _____
e. $40 \div 4 =$ _____

Fluently Multiply and Divide within 100

Lessons

6

9

14

18

Fluency Lessons These lessons are included to reinforce facts learned, to give more time to students who need it to study and practice a fact, and to review strategies.

Independent activities are also included in these lessons.

Independent Activities Students who do not need extra teaching or practice for fluency after completing a Check Sheet may choose from one of the activity options shown below.

► **Go Ahead**

Student Pairs go ahead to the next fact, using the same plan as for 2s and 5s: study, partner check, complete Check Sheet for that fact.

► **Go for Speed**

Student Pairs go for speed. They can use an answer strip from the back of the Student Activity Book or Activity Workbook, or Check Sheet Answer Strips (TRB M7) to complete a Check Sheet again for one of the facts and record the time it takes to complete it. Then complete it again using another answer strip and compare it with the first time.

► **Invent a Game or Play a Game**

Student Pairs can invent a game to learn or practice multiplications and divisions. Students can also play the games *Solve the Stack* and *High Card Wins* introduced in Lesson 13.

► **Write a Word Problem**

Student Pairs can write word problems that can be solved using a multiplication or division they know. Then exchange to solve.

► **Invent Rhymes or Songs**

Student Pairs write rhymes or songs that will help everyone remember the hardest multiplication and division facts and lead the class to practice them.

from THE PROGRESSIONS FOR
THE COMMON CORE STATE
STANDARDS ON OPERATIONS
AND ALGEBRAIC THINKING

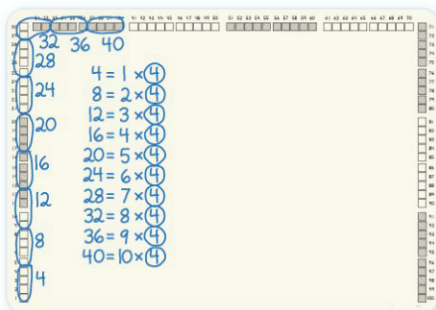
Patterns and Strategies Mastering this material, and reaching fluency in single digit multiplications and related divisions with understanding, may be quite time consuming because there are no general strategies for multiplying or dividing all single-digit numbers as there are for addition and subtraction. Instead, there are many patterns and strategies dependent upon specific numbers. So it is imperative that extra time and support be provided if needed.

Strategies for Multiplying and Dividing

Lessons

1 3 5 7 8 10 11
12 15

Identify and Explain Patterns in Arithmetic Students circle sequential groups of a given number (such as 4) on their Number Path and write the sequential totals. The totals show the multiplication products. Students analyze patterns they see in the count-bys for each number.



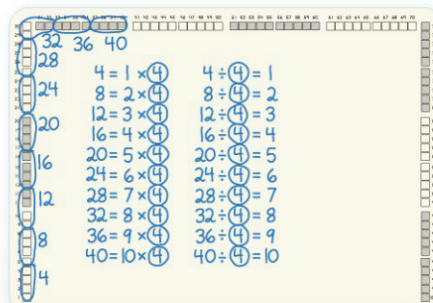
Patterns for 0s and 1s. Students use patterns to make multiplication with 0s and 1s easy. Any number times 0 is 0. Division is not possible with 0. Any number multiplied by 1 is the original number. Any number divided by 1 is the number being divided.

Use the Relationship Between Multiplication and Division Students use their circled sequential groups on the Number Path and their knowledge of multiplication to write the related division equations. By studying the two operations together, students see that doing division is the same as finding an unknown factor in a multiplication situation.

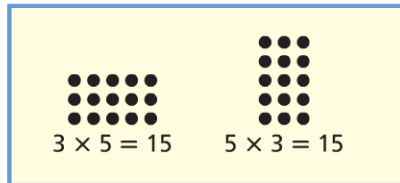
Use Drawings Students use Equal Shares, Equal Groups, and Fast Array drawings to represent known and unknown factors and products in a conceptual format and to write equations and solve problems.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Equal Groups In Equal Groups, the roles of the factors differ. One factor is the number of objects in a group (like any quantity in addition and subtraction situations), and the other is a multiplier that indicates the number of groups. So, for example, 4 groups of 3 objects is arranged differently than 3 groups of 4 objects. Thus there are two kinds of division situations depending on which factor is the unknown (the number of objects in each group or the number of groups).

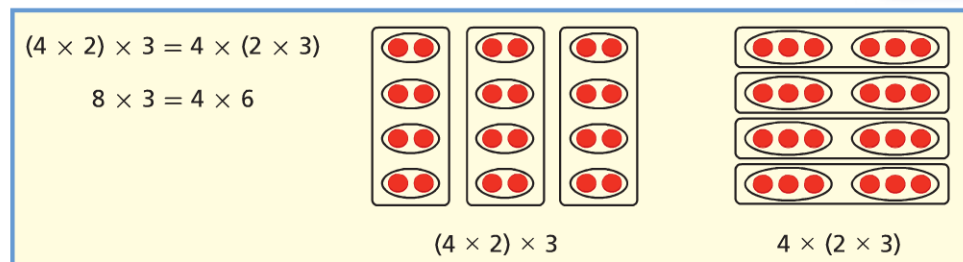


Use Properties of Multiplication The array model for multiplication leads students to understand the Commutative Property of Multiplication. For example, 3 rows of 5 objects results in the same number of objects as 5 rows of 3 objects. This helps students derive new facts from facts they already know.



The Associative and Distributive Properties help students build fluency with multiplication by using facts they know to find unknown products.

The Associative Property allows students to change the grouping of factors presented.



Multiplication and Area The Distributive Property allows students to break apart facts they don't know into known facts by relating area.

$$7 \times 3 =$$

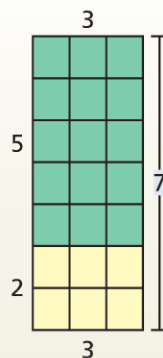
$$(5 + 2) \times 3 = (5 \times 3) + (2 \times 3)$$

Add the areas of the two smaller rectangles.

$$5 \times 3 = 15 \text{ square units}$$

$$2 \times 3 = 6 \text{ square units}$$

$$= 21 \text{ square units}$$



from the PRog Ressions fo R the CoMMon CoRe stAte stAnd ARds on oPeRations And Alge BRAiC t hinking

Arrays In the array situations, the roles of the factors do not differ. One factor tells the number of rows in the array, and the other factor tells the number of columns in the situation. But rows and columns depend on the orientation of the array. If an array is rotated, the rows become columns and the columns become rows. This is useful for seeing the Commutative Property for Multiplication in rectangular arrays and areas.

from the PRog Ressions fo R the CoMMon CoRe stAte stAnd ARds on oPeRations And Alge BRAiC t hinking

Multiplication and Area Area problems where regions are partitioned by unit squares are foundational for Grade 3 standards because area is used as a model for single-digit multiplication and division strategies such as decomposing to find the sum of two known facts.

Represent and Solve Problems Involving Multiplication and Division

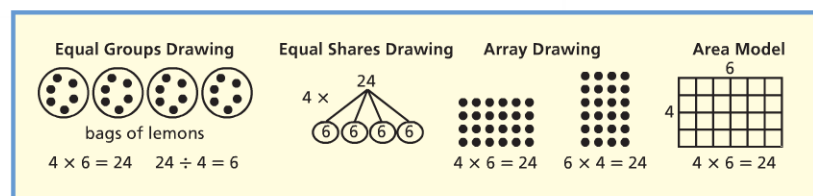
Lessons



In *Math Expressions* a research-based problem solving approach that focuses on problem types is used.

- Interpret the problem
- Represent the situation
- Solve the problem
- Check that the answer makes sense

Students using *Math Expressions* are taught a variety of ways to represent word problems. Some are conceptual in nature (making math drawings), while others are symbolic (writing equations). Students move from using math drawings to solving problems symbolically with equations. The following are math drawings students use to represent multiplication and division word problems in this unit.



Situation and Solution Equations Students are introduced to *situation* and *solution equations* in Lesson 5. Students may represent a word problem with a situation equation. A situation equation shows the order of the information. Then they may rewrite the situation equation as a solution equation. A solution equation shows the operation that can be used to solve the problem.

Situation: Rhonda divided 8 crayons equally between her twin brothers. How many crayons did each boy get?

You might write $8 \div 2 = \square$ as a situation equation, but you would actually think $2 \times \square = 8$ to find the answer. That is your solution equation.

Focus on Mathematical Practices

Lesson



The Standards for Mathematical Practice are included in every lesson of this unit. However, there is an additional lesson that focuses on all eight Mathematical Practices. In this lesson, students describe strategies for multiplying and make a pictograph.

from the Progression S for the Common Core State Standards on Operations and Algebraic Thinking

relating Equal groups and a array Situations Array situations can be seen as Equal Group situations if each row or column is considered as a group. Relating Equal Group situations to Arrays, and indicating rows or columns within arrays, can help students see that a corner object in an array (or a corner square in an area model) is not double counted: at a given time, it is counted as part of a row or as a part of a column but not both.

rows and Columns Row and column language can be difficult. The Array problems are of the simplest form in which a row is a group and Equal Groups language is used ("with 6 apples in each row"). Such problems are a good transition between the Equal Groups and array situations and can support the generalization of the Commutative Property. Problems in terms of "rows" and "columns," e.g., "The apples in the grocery window are in 3 rows and 6 columns," are difficult because of the distinction between the number of things *in* a row and the number *of* rows.