Getting Ready to Teach Unit 4

Learning Path in the Common Core Standards

In this unit, children work toward building fluency with money and subtraction and mastering all addition and subtraction word problem subtypes.

Visual models and real world situations are used throughout the unit to understand subtraction methods and word problems.

Help Children Avoid Common Errors

Math Expressions gives children opportunities to analyze and correct errors, explaining why the reasoning was flawed.

In this unit we use Puzzled Penguin to show typical errors that children make. Children enjoy teaching Puzzled Penguin the correct way, why this way is correct, and why Puzzled Penguin made the error. Common errors are presented in the Puzzled Penguin feature in the following lessons:

- **Lesson 2:** Omitting the zero in money amounts with less than ten cents
- **Lesson 5:** Not ungrouping when there are not enough ones to subtract from
- **Lesson 6:** After ungrouping, forgetting to record the new number of tens before subtracting
- **Lesson 20:** Assuming that *more* always indicates addition in word problems

In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As a part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.

Math Expressions VOCABULARY

As you teach this unit, emphasize understanding of this term.

- ungroup

See the Teacher Glossary.

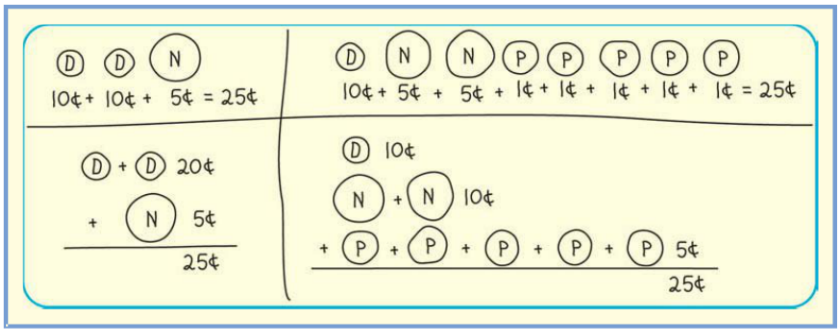
Working with Money Amounts

Lessons 1 2

The Grade 2 Common Core State Standards for Measurement and Data require children to solve word problems involving dollar bills, quarters, dimes, nickels, and pennies. These two lessons support that standard. Children also worked with money amounts in Unit 2.

Understand Money Equivalents Working with money amounts is both a practical skill and a mathematical skill. Although parts of our money system reflect a base ten system (pennies, dimes, and dollars), other parts do not (nickels and quarters). This inconsistency makes working with money confusing for children, especially for children from countries with different money systems.

Finding and learning the coin combinations that are equivalent to a nickel, a dime, a quarter, and a dollar gives children much practice in adding within 100 and is important as a real life skill. The first lesson focuses on equivalents for quarters, and the second lesson covers equivalents for dollars.



Counting Money Amounts When children count money amounts, they draw on their ability to skip count or to add quickly, and they learn to shift the counting amount when counting different coins. At this grade level, children are presented with coins in order from greatest to least value. You might point out that ordering coins before counting makes it easier to count the coins quickly, but do not expect children to order coins before counting them.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

Understand Place Value Students begin to work towards multiplication when they skip count by 5s, by 10s, and by 100s. This skip counting is not yet true multiplication because students don't keep track of the number of groups they have counted.

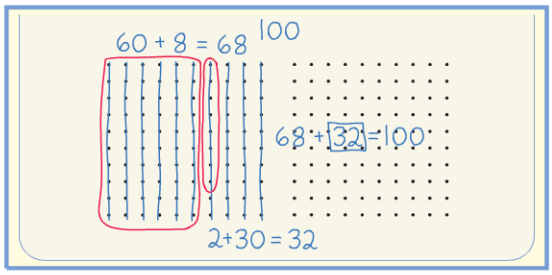
Relating Addends to Subtraction

Lessons 3 4

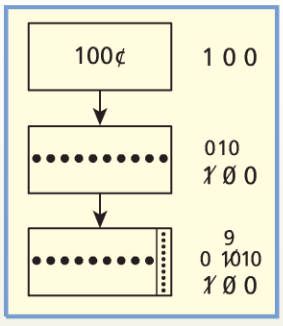
In these two lessons, children use their mathematical understanding of the relationship between finding an unknown addend and subtracting. They may use drawings or methods that they learned in Grade 1. These lessons prepare the children for the more formal introduction of methods of subtracting in Lesson 5.

Ungrouping It is important that children understand why and when they need to ungroup the total (or the larger number) in order to find the unknown addend. If children see that they are ungrouping this number in a different way so that they can subtract, they are less likely later to make the common error of subtracting the smaller digit in a place from the larger digit, without regard to which number is being subtracted.

Subtracting from 100 Children may use the dot array on their MathBoards to model problems that involve subtracting from 100. To find the unknown addend or the difference for 100 − 68, children may make a drawing like the one below. When they use drawing on the dot array, they do not need to ungroup as they are simply breaking 100 apart into two addends.



Or they may use money as a model and ungroup by trading 1 dollar for 10 dimes, and 1 dime for 10 pennies. They may use play money or make drawings. To find 100 − 68, they can ungroup 1 dollar as 9 dimes and 10 pennies, and then take away 6 dimes and 8 pennies to find the difference. Using money as a model helps children see why it is sometimes necessary to ungroup.

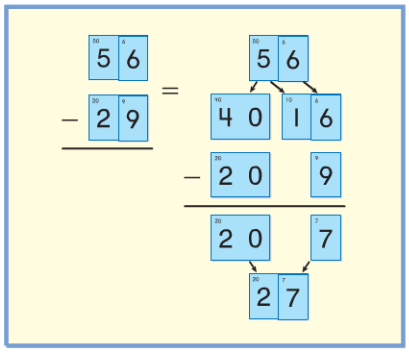


from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

Use place value understanding and properties of operations to add and subtract At Grade 2, composing and decomposing involves an extra layer of complexity beyond that of Grade 1. This complexity manifests itself in two ways. First, students must understand that a hundred is a unit composed of 100 ones, but also that it is composed of 10 tens. Second, there is the possibility that both a ten and a hundred are composed or decomposed. For example, in computing 398 + 7 a new ten and a new hundred are composed. In computing 302 − 184, a ten and a hundred are decomposed.

Subtracting from Other Numbers Children extend their thinking to explore subtracting from numbers other than 100. They again use models to help them find the unknown addend or the difference, depending on how they represent the problem. To find 56 − 29, some children might model 56 on the dot array, circle 29 dots, and count the remaining 27 dots.

The Secret Code Cards are also an effective model for the subtraction process, as shown below for 56 − 29. This model helps children see how the larger number is taken apart in ungrouping and why this makes the subtraction possible.



Deciding When to Ungroup When ungrouping all places of the larger number before beginning to subtract, children are introduced to the idea that it is sometimes helpful to examine the larger number and the number being subtracted to help them decide in what places ungrouping is needed. In Lessons 8 and 9, children will explore this concept in depth as they consider different approaches to ungrouping and subtracting.

***from* THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN**

Use place value understanding and properties of operations to add and subtract Students become fluent in two-digit addition and subtraction. Representations such as manipulative materials and drawings may be used to support reasoning and explanations about addition and subtraction with three-digit numbers.

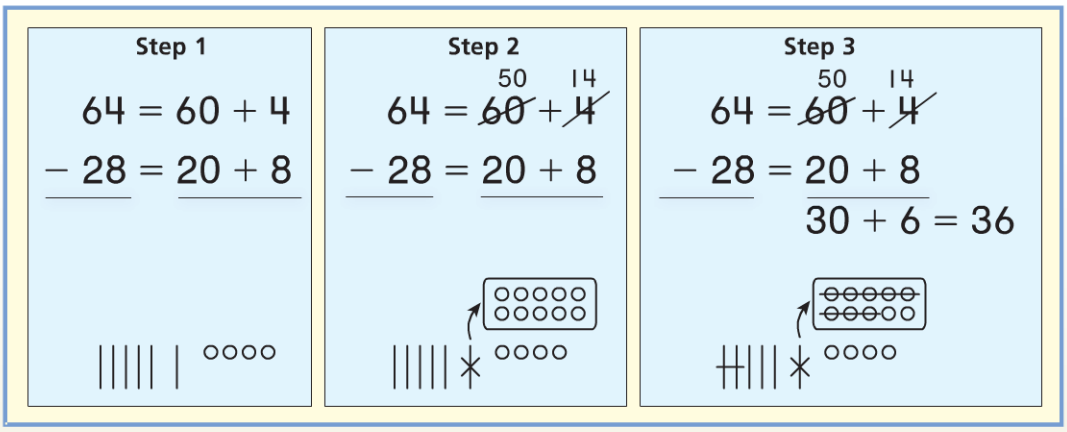
Accessible Subtraction Algorithms

Lesson 5 6 7 11 14

Two research-based subtraction algorithms, the Expanded Method and the Ungroup First Method, are used in *Math Expressions* to help children build a foundation for understanding of and fluency in subtracting. Both are accessible algorithms, as children can understand and explain the steps as they work, and both help children understand why ungrouping the larger number is often the first step in subtracting.

Expanded Method To subtract with this method, children first break apart both numbers by place value. They break apart 2-digit numbers into tens and ones, and they break apart 3-digit numbers into hundreds, tens, and ones. Because this helps them see the real value of each part of the larger number as they ungroup it and then subtract, they come to understand why ungrouping makes it possible to subtract place by place. You might remind children that they used the Show All Totals Method to add in which they found the totals for all places and then grouped the totals to find the answer. Subtracting with the Expanded Method is like working backward with the Show All Totals Method of adding.

As they work with this method, children use proof drawings to verify their work. They use a drawing to show what is happening in the numerical algorithm. This reinforces the concept that the entire smaller number is taken away from the entire larger number and helps children avoid the error of simply finding the difference of the two digits in a place.



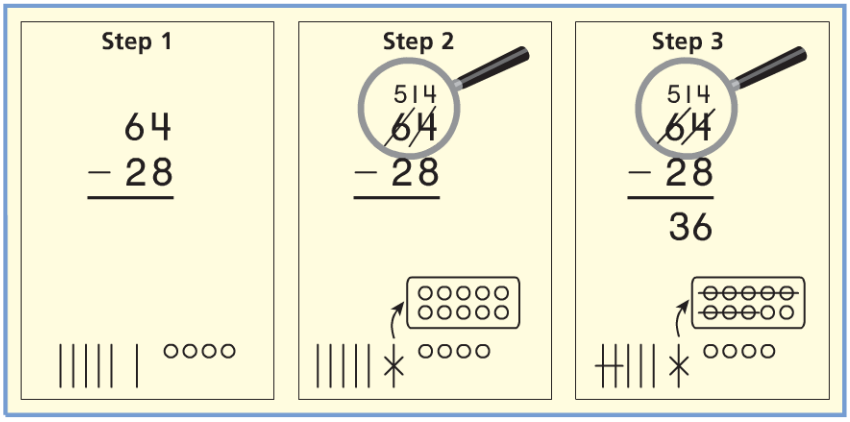
Because this algorithm fosters understanding in such a strong way, it is introduced first. You should let children use it as long as they need it, but encourage them to move to the Ungroup First Method as soon as they no longer need the conceptual support of this method.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

Use place value understanding and properties of operations to add and subtract Students become fluent in two-digit addition and subtraction. … When students add ones to ones, tens to tens, and hundreds to hundreds they are implicitly using a general method based on place value and the associative and commutative properties of addition.

Ungroup First Method To subtract with this method, children work with the number they are subtracting from as a whole. They prepare this number for subtraction by ungrouping in all places. They use a magnifying glass to look inside all of the larger number and see that they are not changing the number to a new amount but are naming it in a new way.

Children also use proof drawings with this method. The drawings give children a visual model of what they are doing in the numerical method. For both methods, the strong conceptual approach tied to the proof drawing models helps children avoid the common error of subtracting the smaller digit from the larger digit in a given place.



Compare Methods As children work with the two methods described above, they will likely decide that they like one or the other method better. In Lesson 6, children discuss the advantages and disadvantages of the two methods and explain their reasons for using the method they choose. Use this opportunity to listen to their explanations and reasoning to help you see how well each child understands the concept of subtracting with 2-digit and 3-digit numbers. In Lesson 7, children have another opportunity to compare the two methods as they work with subtracting from 200.

Work Toward Fluency Throughout the rest of the unit, in practice exercises, a game, and word problems, children are provided with the practice necessary to achieve fluency in subtracting within 100 by the end of Grade 2.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

Use place value understanding and properties of operations to add and subtract Drawings and diagrams can illustrate the reasoning repeated in general methods for computation that are based on place value.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

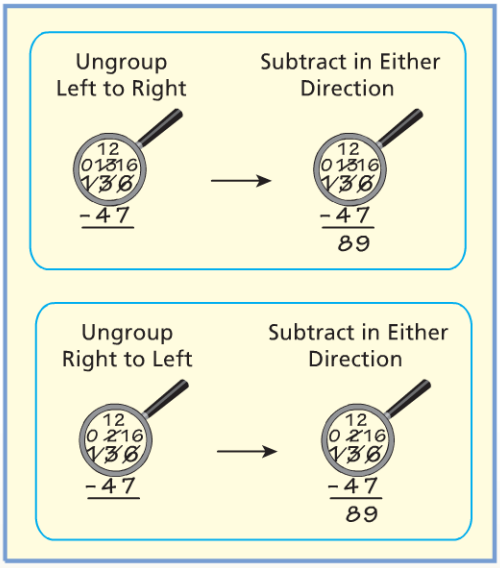
Grade 2 The word *fluent* is used in the Standards to mean "fast and accurate." … It is important to push sensitively and encouragingly toward fluency of the designated numbers at each grade level, recognizing that fluency . . . may differ across students.

Extend Subtraction Concepts

Lessons 8 9 10 13 15

As children become familiar with the two subtraction methods, they are ready to dig deeper into subtraction. They learn ways to vary their approach to the Ungroup First Method, to work with zero in the ones place or the tens place of the larger number, and to model subtraction with money.

Work in Any Direction One strength of the Ungroup First Method is its adaptability. Unlike the standard subtraction algorithm in which ungrouping and subtracting occur alternately in each place, working from right to left, the Ungroup First Method yields the correct answer no matter which direction the ungrouping and the subtraction are done. Because the method assures that the top number in each place will be greater than the bottom number, the subtraction will be correct. As children explore these variations, suggest that they compare their answers to see whether they are the same.



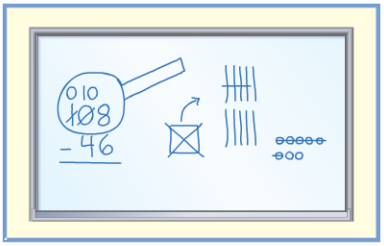
Decide Whether to Ungroup When children use the Ungroup First Method, they need to consider which ungroupings are necessary. Guide them to compare the digits in each place of the two numbers to decide whether they can subtract with those digits or whether they need to ungroup in order to subtract.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

Use place value understanding and properties of operations to add and subtract Students become fluent in two-digit addition and subtraction. Representations such as manipulative materials and drawings may be used to support reasoning and explanations about addition and subtraction with three-digit numbers.

Subtract from Zero In Lesson 9, children use the Ungroup First Method with proof drawings to see that working with a zero in the larger number is not any different from working with other digits. They simply ungroup until the top number in a place is greater than the number to be subtracted in that place. It may be helpful to remind children how they ungrouped when they subtracted from 100 and from 200.

The first subtraction involving zero in the ones place or the tens place is 108 – 46. The illustration below shows the steps and drawings that children would use to do this subtraction. Although they are able to subtract in the ones place, when they get to the tens place, they need to go to the hundreds place and ungroup the 1 hundred as 10 tens. Use of the magnifying glass helps emphasize the importance of working with 108 as a whole. The proof drawing shows how the subtraction takes place with the ungrouped tens. As you have done before, continue to use a horizontal line to subtract so that children see the connection between the subtraction and the subtraction sign.



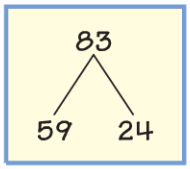
Model Subtraction with Money As reinforcement for the ungrouping concepts in this unit, children use pennies, dimes, and dollars to model transactions at a yard sale. They see that they can subtract with money just as they subtract with numbers.

Ungrouping from the left		Ungrouping from the right
15 0 16 12 1 6 2 ¢ – 8 4 ¢ 7 8 ¢	or	15 0 5 12 1 6 2 ¢ – 8 4 ¢ 7 8 ¢

***from* THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN**

Use place value understanding and properties of operations to add and subtract Subtractions such as 302 –184 can be computed using a general method by decomposing a hundred into 10 tens, then decomposing one of those tens into 10 ones.

Write Equations from Math Mountains Children have used Math Mountains to represent addition and subtraction with 1-digit addends. These number bond diagrams help children see how the addends are related to the sum (or total). In Lesson 13, children discover that they can use Math Mountains with 2-digit addends to represent addition and subtraction situations. In the Math Mountain below, the two addends are 59 and 24 and the sum is 83.



Children also learn that they can use the diagrams to help them write the eight related equations for a given addition or subtraction, just as they did with 1-digit addends. By exploring all the relationships shown in the Math Mountain above, children can generate eight equations.

$59 + 24 = 83$	$83 = 59 + 24$
$24 + 59 = 83$	$83 = 24 + 59$
$83 - 24 = 59$	$59 = 83 - 24$
$83 - 59 = 24$	$24 = 83 - 59$

Adding Up Method In Lesson 15, children apply what they know about counting on to develop the Adding Up Method for finding an unknown addend or a difference. As children discuss ways to add up from 86 to 151, they build on previously learned concepts. They think about efficient ways of counting on, chunking the unknown addend in different ways. Some children may add 6 to 84 to get 90, then add tens to 150, and finally add one to reach 151. Others may add 6 to 84 to get 90, add 10 to get 100, and add 51 to get 151.

Encourage the children in your class to share all their ideas when you work through this lesson by explaining that there are many ways to add up and that all the ways that give a correct answer are good ways to use. If the method a child chooses to use yields the correct answer, then that child should use that method as long as he or she wants to. However, finding a new, more efficient method is always acceptable.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBER AND OPERATIONS IN BASE TEN

Use place value understanding and properties of operations to add and subtract Students could also view it (302 − 184) as an unknown addend problem 184 + ☐ = 302, thus drawing on the relationship between subtraction and addition. With this view, students can solve the problem by adding on to 184: first add 6 to make 190, then add 10 to make 200, next add 100 to make 300, and finally add 2 to make 302. They can then combine what they added on to find the answer to the subtraction problem: 6 + 10 + 100 + 2 = 118. This strategy is especially useful in unknown addend situations. It can be carried out more easily in writing because one does not have to keep track of everything mentally.

Addition and Subtraction Word Problems

Lessons 12 13 14 15 16 17 18 19 20

Addition and subtraction situations usually have three parts, and any one of the three parts may be unknown. In *Math Expressions*, children write an equation to represent the situation in a word problem. You can think of this equation as a situation equation, but it is not necessary for children to use this terminology. Sometimes, as in *Add To* (Result Unknown) situations, the situation equation can be used to solve the problem. For other situations, such as *Compare* (Difference Unknown), children may need to rewrite the situation equation to reflect the operation they will use to find the solution. You can think of this equation as a solution equation.

Problem Types This sequence of lessons focusing on problem solving is designed to give children experience with the various problem types described in the Common Core Standards. The following summary shows the problem types and the kinds of equations for each type that children in Grade 2 should be familiar with and be able to solve and to create problems about. Sample problems for each problem type are given on page T8 of this Teacher's Edition.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Grade 2 Students in Grade 2 master all of the problem situations and all of their subtypes and language variants. The numbers in these problems involve addition and subtraction within 100. They represent these problems with diagrams and/or equations.

Situation	Unknowns		
Add To	Result 3 + 4 = ?	Change 3 + ? = 7	Start ? + 4 = 7
Take From	Result 7 − 3 = ?	Change 7 − ? = 4	Start ? − 3 = 4
Put Together/Take Apart	Total 4 + 3 = ?	Addend 4 + ? = 7 or 7 − 4 = ?	Both Addends 7 = ? + ?
Compare	Greater Number 3 + ? = 7 or 7 − 3 = ?	Greater Number 3 + 4 = ? or 4 + 3 = ?	Lesser Number 7 − 4 = ? or 4 + ? = 7