Getting Ready to Teach Unit 1

Learning Path in the Common Core Standards

In this unit, children work toward building fluency with addition and subtraction within 20 and mastering all addition and subtraction word problem subtypes.

Visual models and real world situations are used throughout the unit to help children understand the meaning of addition and subtraction.

Help Children Avoid Common Errors

Math Expressions gives children opportunities to analyze and correct errors, explaining why the reasoning was flawed.

In this unit we use Puzzled Penguin to show typical errors that children make. Children enjoy teaching Puzzled Penguin the correct way, why this way is correct, and why Puzzled Penguin made the error. Common errors are presented in the Puzzled Penguin feature in the following lessons:

- **Lesson 6:** Eliminating decade numbers when counting by 2s
- **Lesson 8:** Misunderstanding the meaning of the equal sign when the expression after the equal sign includes more than one number and an operation sign
- **Lesson 15:** Incorrectly labeling comparison bars
- **Lesson 20:** Not recognizing extra information in a word problem

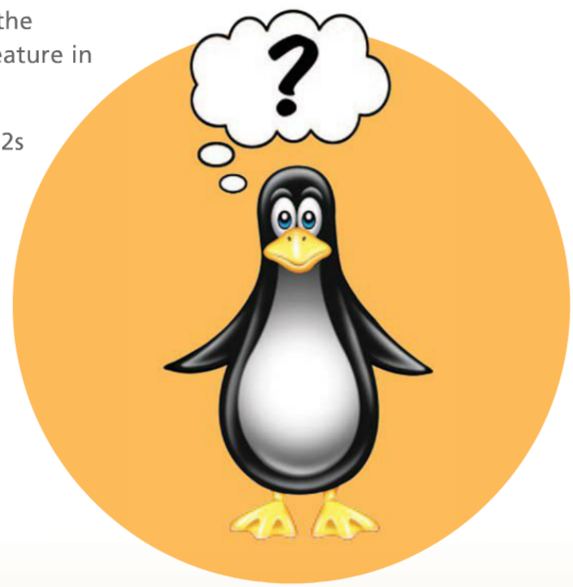
In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As a part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.

***Math Expressions* VOCABULARY**

As you teach this unit, emphasize understanding of these terms.

- partner
- Math Mountain
- teen number
- equation chain

See the Teacher Glossary.

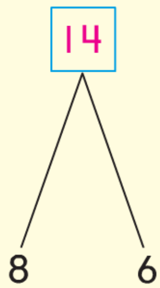


Relate Addition and Subtraction

Lessons **1** **2**

Math Mountains and Equations Math Mountains are used in *Math Expressions* to show how addition and subtraction are related. A Math Mountain shows a total on top and two partners (addends) at the bottom. In Lesson 1, children relate Math Mountains to addition and subtraction equations and to real world problems.

Math Mountain



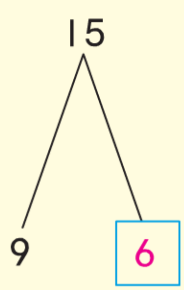
Equation

$$8 + 6 = \boxed{14}$$

Real World Problem

There are 8 flowers in a vase. There are 6 flowers in a glass. How many flowers are there altogether?

Math Mountain



Equations

$$9 + \boxed{6} = 15$$
$$15 - 9 = \boxed{6}$$

Real World Problems

There were 9 children playing in the park. Some more children came. Now there are 15 children playing. How many children came to the park?

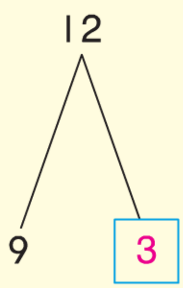
There were 15 children playing in the park. Nine went home. How many children are still playing?

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

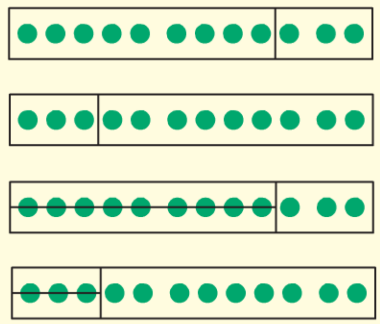
Relate Addition and Subtraction

Diagrams used in Grade 1 to show how quantities in the situation are related continue to be useful in Grade 2, and students continue to relate the diagrams to situation equations. Such relating helps students rewrite a situation equation like $\square - 38 = 49$ as $49 + 38 = \square$ because they see that the first number in the subtraction equation is the total. Each addition and subtraction equation has seven related equations. Students can write all of these equations, continuing to connect addition and subtraction, and their experience with equations of various forms.

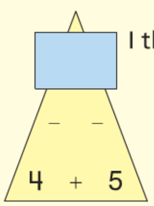
Related Equations Children discuss why the eight equations below come from the Math Mountain. In their discussion, they informally discuss properties of addition and of equality, and the relationship between addition and subtraction.



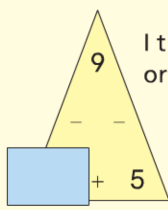
$9 + 3 = 12$		$12 = 9 + 3$
$3 + 9 = 12$		$12 = 3 + 9$
$12 - 9 = 3$		$3 = 12 - 9$
$12 - 3 = 9$		$9 = 12 - 3$



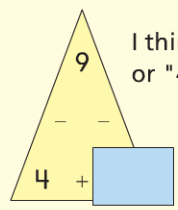
Math Mountain Cards Children use Math Mountain Cards to practice addition and subtraction. The cards make clear how addition, subtraction, and finding an unknown addend are related. The cards reinforce that the same process is used for subtraction and for finding an unknown addend.



I think: "4 + 5 = 9."



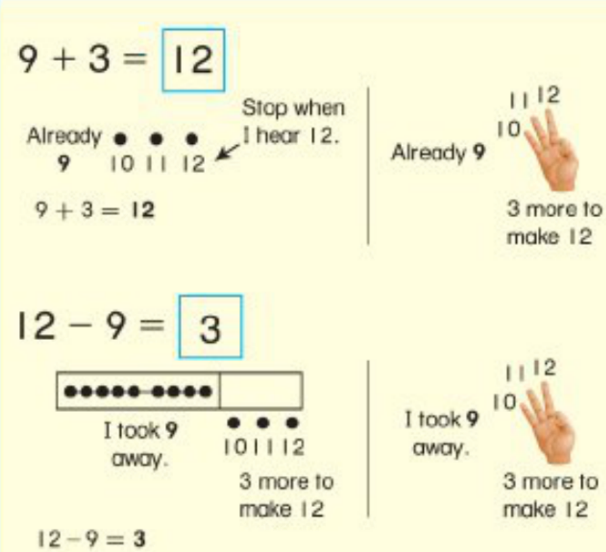
I think: "9 – 5 = ?"
or "5 + ? = 9."



I think: "9 – 4 = ?"
or "4 + ? = 9."

Count On for Addition or Subtraction Children at Level 1 for addition *count all;* children at Level 1 for subtraction *take away.* Children at Level 2 *count on.* At Grade 2, children should be using either Level 2 or Level 3 strategies for addition and subtraction. Level 2 strategies are reviewed in Lesson 1. Level 3 strategies are reviewed in Lessons 3–5. (At Level 3, children decompose an addend and compose a part with another addend.)

As the examples below illustrate, the process for counting on to find a total and counting on to find a partner look the same. Only the solver knows which problem is being solved.



In counting on to find the **total 12,** you keep track of the second addend 3 and stop when your fingers show 3. The words tell the total **12**.

In counting on to find the **unknown addend 3,** you keep track of the words you say and stop when you hear 12. The number of fingers tell the unknown addend **3.**

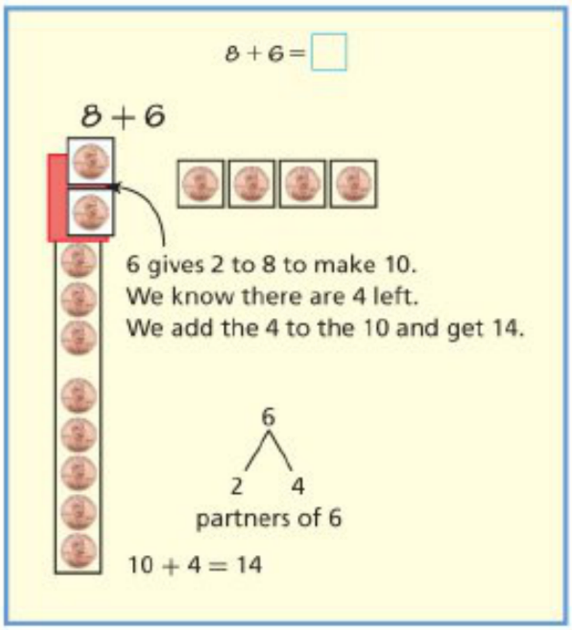
Using the Commutative Property of Addition The Commutative Property of Addition states that two addends can be added in either order and the sum remains the same. So, for addition, children could begin counting on from either addend. However, children should be encouraged to count on from the greater addend. In this way, there will be fewer numbers to count on and less chance of errors.

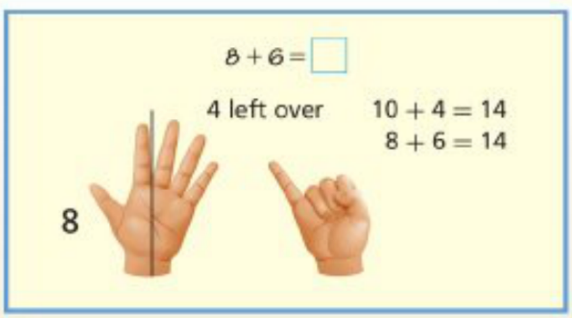
Make-a-Ten Strategies

Lessons 3 4 5

At Level 3, children decompose an addend and compose a part with another addend. Make a Ten is a Level 3 strategy. The goal at Grade 2 is for children to be using mental Level 2 and Level 3 strategies.

Make-a-Ten Strategy for Addition The difficult part of this strategy is separating the smaller addend into two parts: the amount that when added to the greater addend makes 10 and "the rest." Children model this strategy using coin strips and fingers.





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Addition and Subtraction Within 20

The deep extended experiences students have with addition and subtraction in Kindergarten and Grade 1 culminate in Grade 2 with students becoming fluent in single-digit additions and the related subtractions using the mental Level 2 and 3 strategies as needed. So fluency in adding and subtracting single-digit numbers has progressed from numbers within 5 in Kindergarten to within 10 in Grade 1 to within 20 in Grade 2. The methods have also become more advanced.

Make-a-Ten Strategy for Subtraction The Make-a-Ten strategy for subtraction is the same as for finding an unknown addend. While the problem situations and the equations are different, in both problems, the total is known and one addend (or partner) is unknown.

$8 + \square = 14$

Step 1

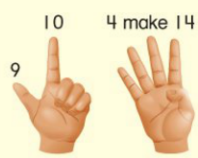
Think: I already have 8.

Step 2

Put up 2 to make 10.

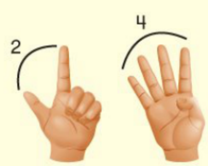
Step 3

Put up 4 more to make 14.



Step 4

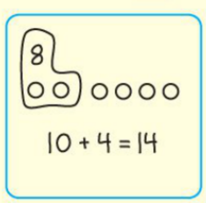
Find the unknown partner.



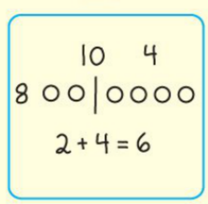
$2 + 4 = 6$

Using Drawings for the Make-a-Ten Strategy Children use drawings to show the Make-a-Ten strategy. Children may generate any of a variety of drawings to illustrate this strategy. Two samples are shown below.

$8 + 6 = \square$



$8 + \square = 14$



Even and Odd Numbers

Lesson 6

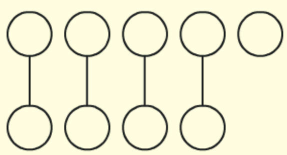
Children explore several methods for deciding whether a number is even or odd. In this **Math Talk in Action,** children share those methods.

MATH TALK in ACTION Below is a sample classroom discussion of the different ways to determine whether 9 is even or odd.

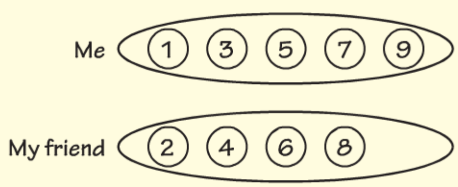
Let's talk about how we can decide whether 9 is an even number or an odd number.

Allison: I know 9 is odd. When I count by 2s, I skip right over it: 2, 4, 6, 8, . . . 10. I went right from 8 to 10. When we count by 2s, we say the even numbers.

Marcus: It's odd for sure. When I draw pairs of counters, I can't make 9 without having one counter not in a pair.



Jorge: I know another way to show 9 is odd. Suppose I have 9 marbles. I can't share 9 marbles equally with my friend. One of us will get one more marble than the other. I can draw a picture to show what I mean.



You just can't share 9 marbles so it's fair or even. So 9 must be odd.

Darius: If 9 is even, we could write an addition double for it. I know 4 + 4 = 8 and 5 + 5 = 10. There is no addition double for 9. So, 9 has to be odd.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Odd and Even Numbers Standard 2.OA.3 relates doubles additions up to 20 to the concept of odd and even numbers and to counting by 2s by pairing and counting by 2s the things in each addend.

Strategies Using Doubles

Lesson **7**

Doubles Plus/Minus 1 Children can use either the Doubles Plus 1 or the Doubles Minus 1 strategy to add two numbers that are 1 apart.

$7 + 6 = \square$

Using a Double Plus 1	or	Using a Double Minus 1
6 + 6 = 12, so		7 + 7 = 14, so
7 + 6 = 13, 1 more than 12		7 + 6 = 13, 1 less than 14

Doubles Plus/Minus 2 Children can use either the Doubles Plus 2 or the Doubles Minus 2 strategy to add two numbers that are 2 apart.

$8 + 6 = \square$

Using a Double Plus 2	or	Using a Double Minus 2
6 + 6 = 12, so		8 + 8 = 16, so
8 + 6 = 14, 2 more than 12		8 + 6 = 14, 2 less than 16

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Doubles Strategies Another Level 3 method that works for certain numbers is a doubles ±1 or ± 2 method:
$6 + 7 = 6 + (6 + 1) = (6 + 6) + 1$
$= 12 + 1 = 13.$
These methods do not connect with place value the way make-a-ten methods do.

Equations, Equation Chains, and Vertical Form

Lesson 8

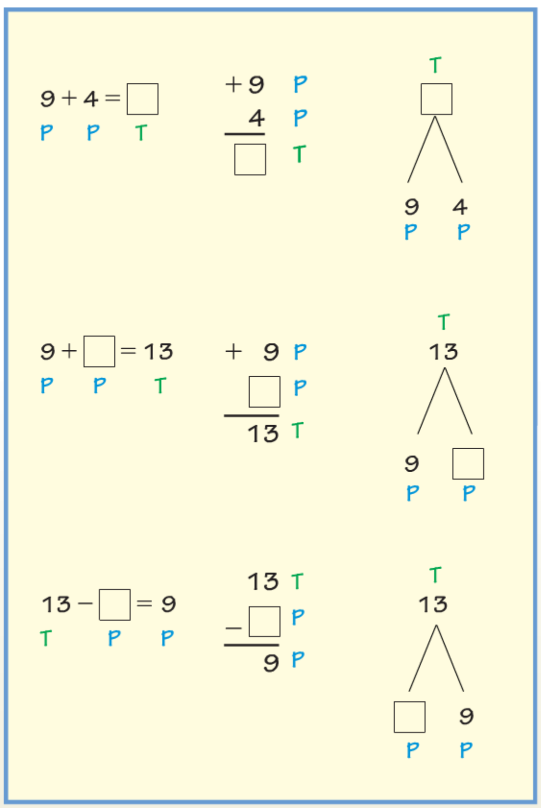
Equations Children work with equations that have operations on both sides of the equal sign, for example, $6 - 1 = 4 + 1$. This reinforces that = means "is the same as," not "makes" or "results in."

Equation Chains Children write equation chains. Equations chains have more than one equal sign. To help children see the equivalent expressions, they loop them within an equation chain.

$$5 = 2 + 3 = 3 + 2 = 1 + 4 = 4 + 1 = 6 - 1$$

Equation chains are used in Quick Practices to help children generate several addition and subtraction expressions for a number.

Vertical Form Children rewrite equations in vertical form. Children label the partners (P) and the total (T) to connect the two forms.



from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Equations Equations with totals on the left help children understand that = does not always mean "makes" or "results in" but always means "is the same number as."

Add Three or Four Addends

Lesson **9**

Using the Associative Property of Addition When adding three or four addends, children develop their own strategies for grouping the addends to make the addition easier. They make use of place value (making one or more tens) and the Associative Property of Addition, which states that addends can be grouped in any way and the sum stays the same. By sharing and discussing their strategies, children develop effective and efficient strategies for adding three or more numbers.

> MATH TALK *in ACTION* Again invite children who used different methods to share them.
>
> **Who would like to share their method for finding 8 + 8 + 5 + 2?**
>
> *Carl:* I added the 8 and 2 and got 10. Then I added the 8 and 5 and got 13. To add 10 and 13, I just thought about 10 + 10 (which is 20) and 3 more, which is 23.
>
> *Isabella:* I added 8 + 8 and got 16. Then I broke the 5 into 4 + 1 so I could use the 4 to make 20. Then I just added in the 1 and the 2. So, my answer is the same, 23.
>
> *Aiden:* I added 8 + 8 and got 16. Then I added 5 + 2 and got 7. I didn't know what 16 + 7 was, so I counted on from 16. *(Uses fingers to keep track of 7 numbers.)* 17, 18, 19, 20, 21, 22, 23. The total is 23.

Adding three or four 1-digit numbers prepares children for adding up to four 2-digit numbers. [CC.2.NBT.6]

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON NUMBERS AND OPERATIONS IN BASE TEN

Fluency with Place Value and Properties Both general methods and special strategies are opportunities to develop competencies relevant to the NBT standards. Use and discussion of both types of strategies offer opportunities for developing fluency with place value and properties of operations, and to use these in justifying the correctness of computations (MP.3).

Add To and *Take From* Word Problems

Lessons **10** **11**

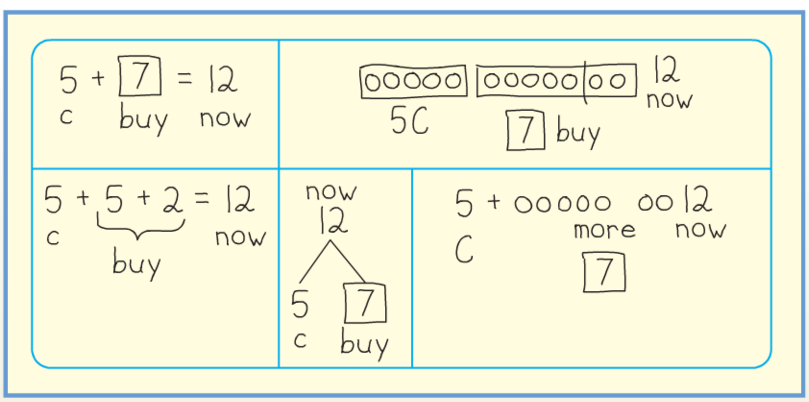
The *Math Expressions* program teaches children to solve word problems using a learning progression that was developed through classroom research during the *Children's Math Worlds* NSF-funded research project headed by the author.

See page T8 of this Teacher's Edition for a summary of all the problem types children need to master in Grade 2.

***Add To* and *Take From* Problems** In Lesson 10, children represent and discuss solution methods for solving these problems. In Lesson 11, they extend their work to include writing their own word problems.

Add To and *Take From* problems start with a quantity that is then modified by a change—something added to or taken from the original quantity—that results in a new quantity. In such problems, the start, the change, or the result may be unknown, so there are six subtypes: unknown start, unknown change, and unknown result for both *Add To* and *Take From* situations. Children apply reasoning or diagrams they have learned for adding and subtracting to solve these problems. The work done with these problem types in Grade 1 provides a solid foundation for continuing and extending problem solving in Grade 2.

Using the **Solve and Discuss** participant structure to work through problems is an excellent way for children to see various ways to solve these problems and for you to see how well children can represent and solve the problems.



from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Grade 2 Grade 2 students build upon their work in Grade 1 in two major ways. They represent and solve situational problems of all three types, which involve addition and subtraction within 100 rather than within 20.

Put Together/Take Apart Word Problems

Lessons 12 13

In *Math Expressions,* children solve word problems by understanding the problem situation so that they can decide the best way to solve the problem. Rather than using word clues or arbitrary rules, they first find a way to understand and represent the problem situation. This gives them a basis for deciding how to solve the problem.

***Put Together/Take Apart* Problems** In Lesson 12, children represent and discuss solution methods for solving these problems. In Lesson 13, they extend their work to include problems with group names and problems that have two unknown addends.

In *Put Together/Take Apart* problems, all objects are present from the start, and nothing is introduced or taken away. The situations involve describing groupings within the total or conceptually putting objects together or taking them apart. In such problems, the total, one addend, or two addends may be unknown, so there are three subtypes: unknown total, unknown addend, and two unknown addends.

In Lesson 13, children work with problems where the two addends may have different names but belong to the same group. Part of solving these problems involves deciding what the group is. Children have been encouraged to use labels in the problem solving lessons in this unit; as they encounter more complex problems, this practice becomes even more important.

In some problems, children are guided to make up sets of objects for a problem situation, as shown in the example below. Note that *apples* and *pears* are subsets of the set *pieces of fruit.*

***from* THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING**

Grade 2 Diagrams used in Grade 1 to show how quantities in the situation are related continue to be useful in Grade 2, and students continue to relate the diagrams to situation equations.

► You Decide

Complete this problem.

Answers will vary.

4. Jenna has 4 apples and Bill has 6 pears. How many pieces of fruit do they have altogether?

o o o o + o o o o o o
A P
4 + 6 = 10

10 pieces of fruit
label

Compare Word Problems

Lessons 14 15 16 18

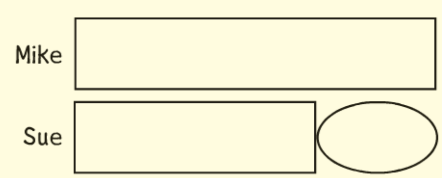
In *Math Expressions,* children use what they know from reading and talking about a problem to draw and label representational diagrams, Math Mountains, number boxes, equations, and comparison bars to solve various types of word problems.

***Compare* Problems** In Lessons 14, 15, and 18, children work with *Compare* problems, at first finding ways to represent these problems and then paraphrasing problem situations and finally working with more complex *Compare* situations involving comparisons in which the language used is opposite to the operation required. In Lesson 16, children work with the various problem types presented in the previous six lessons.

Compare problems involve finding how many more or less one quantity is than another quantity. Children draw number boxes or comparison bars and match objects between groups to help them compare. Children need practice with the more difficult comparison language by saying both forms of a comparison question and using equalizing language:

- How many *fewer* balloons does Sue have than Mike?
- How many *more* balloons does Mike have than Sue?
- How many balloons does Mike need to give away to have *as many as* Sue?

One of the difficulties children may experience with comparison bars is thinking that they need to draw the bars to reflect the problem situation exactly. Help children see that they can draw the comparison bars first and then use the information from the problem to label the bars and fill in the numbers they know. So, for a problem in which Sue has 5 balloons and Mike has 7 balloons, children would start by labeling the longer bar *Mike* and the shorter bar *Sue.*



Then they add the information they know to the diagram and place a question mark to show the information they need to find out.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Some textbooks represent all Compare problems with a subtraction equation, but that is not how many students think of the subtypes. Students represent Compare situations in different ways, often as an unknown addend problem. If textbooks and teachers model representations of or solution methods for Compare problems, these should reflect the variability students show. In all mathematical problem solving, what matters is the explanation a student gives to relate a representation to a context, and not the representation separated from its context.

Find Appropriate Information in Word Problems

Lesson **17**

In real world situations, problems do not have all the required information neatly organized in two or three sentences. Sometimes more information is given than is needed; sometimes not enough information is available, and sometimes information is hidden within the problem.

Problems with Too Much Information Because *Math Expressions* emphasizes understanding the problem situation before trying to solve a problem, children learn to recognize what information is important and what is not needed. For example, a problem may tell how many markers and crayons Shanna has and ask how many markers she has left after giving some to a friend. Children who understand the problem situation know that they do not need to know how many crayons Shanna has to solve the problem and so will cross out that information.

Problems with Not Enough Information Again the emphasis on understanding a problem situation helps children recognize when some information needed to solve a problem is not given in the problem. For example, knowing that Sam and his dog walked 15 blocks in two trips is not enough information to decide how many blocks they walked on one of the trips. Children may need to supply information or say that the problem cannot be solved.

Problems with Hidden Information Some problems have information embedded in quantity words, such as *dozen, week,* or *pair.* Children need to translate the quantity into a number and write it in the problem near the word for the quantity. For example, to solve the following problem, children must understand that there are 3 children in a set of triplets.

13. There are 9 children and a set of triplets in the library. How many children are in the library?

library

Triplet means 3.

12 children

label

Two-Step Word Problems

Lessons 19 20

Read Two-Step Problems Solving a variety of types of two-step problems is a strong focus of the Grade 2 Operations and Algebraic Thinking domain. Because two-step word problems require two steps to solve, it is useful for children to read and rephrase the word problem. As they do, they ask themselves, "What question does this problem want me to answer?" and "What do I need to know first to answer the question?"

Represent Two-Step Problems Children may use a combination of drawings, Math Mountains, and equations to represent the steps of two-step problems. The first step of a problem may be more easily represented with a drawing or Math Mountain that helps children find the information needed to proceed with the second step. Once the information is determined, the second step may often easily be represented with an equation. However, do not push children to use a representation they are not comfortable with.

Solve Two-Step Problems Using the Solve and Discuss structure for these problems provides exposure for all children to different ways of thinking about, representing, and solving two-step problems. As with all problem solving, it may be helpful to ask children to explain why an answer is reasonable or to rephrase the problem situation using the answer to see if it makes sense.

Distinguish Among Problem Types Lesson 20 provides practice with a variety of problem types to help children recognize the problem types and decide how best to represent and solve each type. Note that it is not necessary that children use the words from the Common Core State Standards document to identify problem types but rather they recognize the different types of situations and find ways to solve the problems.

Focus on Mathematical Practices

Lesson 21

The Standards for Mathematical Practice are included in every lesson of this unit. However, the last lesson in every unit focuses on all eight Mathematical Practices. In this lesson, children use what they have learned about adding and subtracting to solve problems about healthy foods.

from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Grade 2 Most two-step problems made from two easy subtypes are easy to represent with an equation, …. But problems involving a comparison or two middle difficulty subtypes may be difficult to represent with a single equation and may be better represented by successive drawings or some combination of a diagram for one step and an equation for the other …. Students can make up any kinds of two-step problems and share them for solving.