Getting Ready to Teach Unit 3

Learning Path in the Common Core Standards

In Unit 1, children were introduced to 1-more and 1-less patterns for addition and subtraction. In Unit 2, children developed strategies for addition and subtraction and discussed different types of equations. This unit focuses on unknown partners represented as both addition and subtraction situations. Children adapt strategies for finding an unknown total to finding an unknown partner. Children write both equations and answers with labels for word problems. Fluency practice within 10 continues in this unit and throughout the rest of the book. In the next unit, children will begin to explore addition and subtraction within 20.

Help Children Avoid Common Errors

Math Expressions gives children opportunities to analyze and correct errors, explaining why the reasoning was flawed.

In this unit we use Puzzled Penguin to show typical errors that children make. Children enjoy teaching Puzzled Penguin the correct way, why this way is correct, and why Puzzled Penguin made an error. Common errors are presented in the Puzzled Penguin feature in the following lessons:

▸ **Lesson 3:** Adds the partner and the total to find the unknown partner.

▸ **Lesson 8:** Mistakes the total for the unknown partner.

▸ **Lesson 10:** Adds the total and one of the partners to find the unknown partner.

In addition to Puzzled Penguin, there are other suggestions listed in the Teacher Edition to help you watch for situations that may lead to common errors. As a part of the Unit Test Teacher Edition pages, you will find a common error and prescription listed for each test item.

Math Expressions **VOCABULARY**

As you teach this unit, emphasize understanding of these terms.
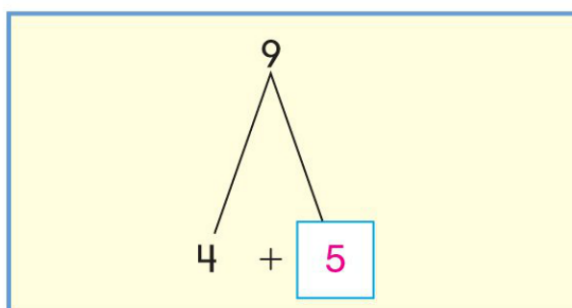
- known partner
- unknown partner

See the Teacher Glossary.
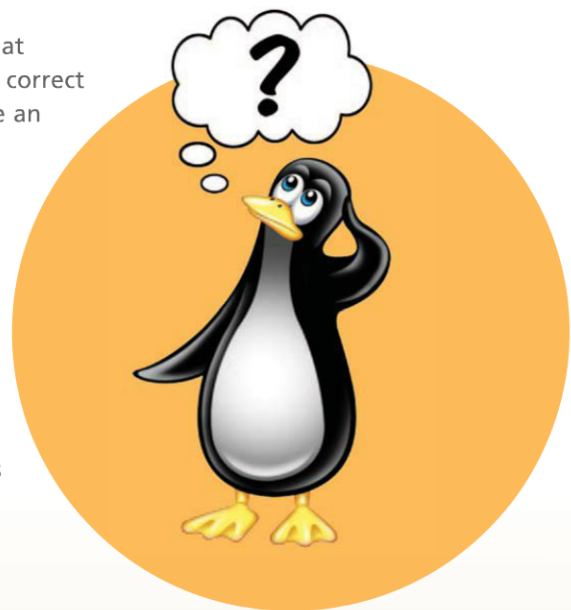
Unknown Partners with Addition

Math Mountains In previous units, children learned how to solve problems to find an unknown total. In Lesson 1, they will use a familiar model to find an unknown partner.

This program clarifies the process by using Math Mountains. In this graphic, the total sits at the top of the mountain and the two partners sit at the base.


Children can relate to this representation quickly because they have used this model in previous units.

Counting On Children learn that one way to find the unknown partner is to count on. They can draw one circle for each number as they count on. They also learn to use their fingers as they count on mentally.

3 more to 7

Two Unknown Partners Children revisit the idea that a total can have different partners. Children develop strategies to find different partners.
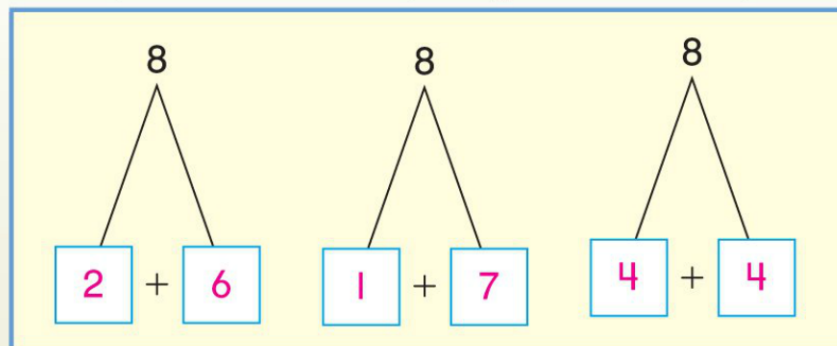
from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Counting On Counting on should be seen as a thinking strategy, not a rote method. It involves seeing the first addend as embedded in the total, and it involves a conceptual interplay between counting and the cardinality in the first addend (shifting from the cardinal meaning of the first addend to the counting meaning). Finally, there is a level of abstraction involved in counting on, because students are counting the words rather than objects. Number words have become objects to students.

Addition with Unknown Partners

Story Problems Children have been telling stories based on the numbers in a Math Mountain since Kindergarten. Now written story problems are introduced in Lesson 2. Children are presented with a story problem.

> I see 6 butterflies.
> 4 are yellow.
> The rest are white.
> How many butterflies are white?

Then, they are asked to find the answer any way they can. Some possibilities are:

Math Mountain   Circle Drawing
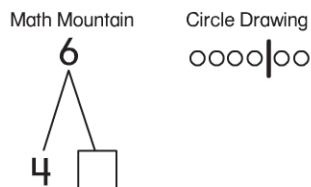
Label the Answer** Children read the problem together as a class. Then children solve the problem and write the answer with a label. This is the first time that children write the label.

> 8 boys are riding bikes.
> 6 ride fast. The rest ride slow.
> How many boys ride slow?
>
> 2   boys
>   label

from **THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING**

Addition Situations Students act out adding and subtracting situations by representing quantities in the situation with objects, their fingers, and math drawings.

To do this, students must mathematize a real-world situation rather than non-mathematical aspects of the situation. Situations can be acted out and/or presented with pictures or words. Math drawings facilitate reflection and discussion because they remain after the problem is solved.
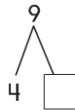
Equations with Unknown Partners

Reading and Solving Equations Children compare Math Mountains and equations that have one unknown partner.
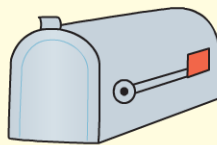
$$4 + \square = 9$$

Children are reminded that they can use the same solution methods for equations as they did for Math Mountains. They are also reminded that they are counting on to find an unknown partner and not counting on to find the total.

Story Problems with Unknown Partners Children discuss what the problem means.

6 letters total

How many letters are in the box?   2   letters
                                              label

Children write an equation and count on to solve it.

$$4 + \square = 6$$
$$4 + \boxed{2} = 6$$

Then they write the answer to the problem.

> **from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING**
>
> **Unknown Addend** These are the situations that can be represented by an addition equation with one unknown addend, e.g., $9 + \square = 13$. Students can solve some unknown addend problems by trial and error or by knowing the relevant decomposition of the total. But a Level 2 counting on solution involves seeing the 9 as part of 13, and understanding that counting the 9 things can be "taken as done" if we begin the count from 9.

Practice with Unknown Partners

Games Children play familiar games that they used to practice unknown totals. In these lessons they will play the same games.

Pancake Breakfast In this variation of the Pancake Breakfast scenario, children find unknown partners.

- *First child:* I made 4 pancakes.
- *Second child:* I made some pancakes, too.
- *First child:* Together we made 6 pancakes.
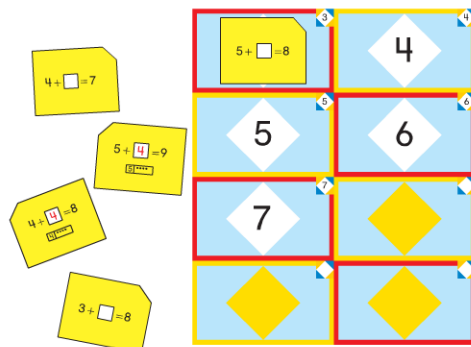- *Second child:* How many pancakes did I make?

$$4 + \square = 6$$
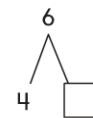
Number Quilt When children played *The Number Quilt Game* in Unit 2, they were counting on to find the unknown total. In this lesson, they will use the Yellow Count-On Cards and Number Quilt 2 to find unknown partners.

Number Quilt 2: Unknown Partners

Number Quilt 2
(Student Activity Book page 73)

Number Grabber One child writes all the Math Mountains for a given total. Another child erases one of the partner numbers on any mountain while the rest of the class closes their eyes. When the Number Grabber says, "Open your eyes for a big surprise," the class looks for the mountain with the unknown partner and then solves for that unknown number.

Story Problems Children practice story problems with an unknown partner. They may discuss counting on using fingers or drawing Math Mountains and circle drawings to solve.
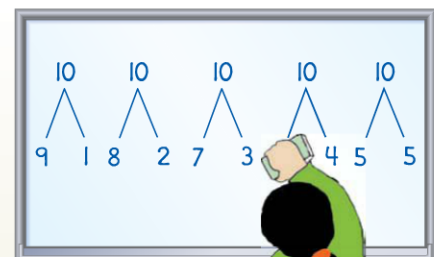
Subtraction with Unknown Partners

Counting On Subtraction problems and problems with unknown partners are solved in the same way because they each require an answer to the question, "How many more do I need to reach the total?" In both types of problems, children count on from the partner, to the total they know, to find the unknown partner.

Finding an Unknown Partner

Unknown Addend

$5 + \boxed{} = 9$

6 7 8 9
● ● ● ●   ┐ 4 more to 9

$5 + \boxed{4} = 9$

Subtraction

$9 - 5 = \boxed{}$

6 7 8 9
● ● ● ●   ┐ 4 more to 9

$9 - 5 = \boxed{4}$

5

8   9 ← Count on from 5 until you reach 9.
7
6

See how many more fingers you needed to make 9.

$5 + \boxed{4} = 9$

Finger Method

Story Problems Children solve and write subtraction story problems.

I draw 10 houses.
Then I erase 5 of them.
How many houses are left?

$\boxed{5}$ _____ houses
label

Games In Lesson 7, children play subtraction games.

Pancake Breakfast Children act out subtraction stories and find the unknown partner by counting on.

Number Quilt Children use a set of Orange Count-On Cards. One side of the card shows a subtraction equation, and the other side shows the answer and a counting-on drawing.

← front
$8 - 5 = \boxed{}$
$8 - 5 = \boxed{3}$ ← back
5 •••

Orange Count-On Cards

Counting On Counting on can be used to add (find a total) or subtract (find an unknown addend). To an observer watching the student, adding and subtracting look the same. Whether the problem is $9 + 4$ or $13 - 9$, we will hear the student say the same thing: "Niiiiine, ten, eleven, twelve, thirteen" with four head bobs or four fingers unfolding. The differences are in what is being monitored to know when to stop, and what gives the answer.

Solving Story Problems

Situation and Solution Equations Children are introduced to situation and solution equations. A *situation equation* matches the structure of the problem. $7 - 3 = \square$ is a situation equation. If a child knows the answer to the subtraction, then it is also a solution equation. A *solution equation* shows the operation that solves a problem.

There are 7 cats. 3 cats walk away. How many cats are left?

$$7 - 3 = \boxed{4}$$

$$3 + \boxed{4} = 7$$

If a child prefers to solve a subtraction problem using addition, then $3 + \square = 7$ might be the better equation to use. In this case, $7 - 3 = \square$ is the situation equation, and $3 + \square = 7$ is the solution equation because it is used to solve the problem.

Relate Addition and Subtraction Completing both a subtraction and addition equation for the same problem emphasizes the relationship between addition and subtraction.

$$8 - \square = 6 \qquad \square - 4 = 5$$

$$6 + \square = 8 \qquad 5 + 4 = \square$$

Children begin to see that they are writing the same unknown in both equations. You can use this opportunity to point out that addition and subtraction are related operations. In later grades, children will come to use the words *inverse operations* to describe this relationship.

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from **THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING**

Situation and Solution Equations Learning where the total is in addition equations (alone on one side of the equal sign) and in subtraction equations (to the left of the minus sign) helps students move from a situation equation to a related solution equation.

from **THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING**

Put Together/Take Apart Problems with Addend Unknown These problems afford students the opportunity to see subtraction as the opposite of addition in a different way than as reversing the action, namely as finding an unknown addend. The meaning of subtraction as an unknown-addend addition problem is one of the essential understandings students will need in middle school in order to extend arithmetic to negative rational numbers.

The Problem Solving Process

Using the Mathematical Practices

Throughout the program, *Math Expressions* integrates a research-based, algebraic problem-solving approach that focuses on problem types. Problem solving is a complex process that involves all eight of the CCSS Mathematical Practices. It is also an individual process that can vary considerably from one child to another. Children may conceptualize, represent, and explain a given problem in different correct ways. This is especially true for additive comparison problems where children may easily reverse the relationships, especially for misleading language problems. Making a drawing helps children show which quantity is larger and then decide on a correct solution equation or computation. Children often prefer to show an addition equation (Smaller + Difference = Larger) rather than the subtraction equation shown in some books as the only representation for comparison situations.

Mathematical Process	Student Actions
Understand the Problem Situation MP.1 Make sense of the problem. MP.2 Reason abstractly and quantitatively.	**Make Sense of the Language** Children use the problem language to conceptualize the real world situation.
Represent the Problem Situation MP.4 Model with mathematics. MP.7 Look for and make use of structure.	**Mathematize the Situation** Children focus on the mathematical aspects of the situation and make a math drawing and/or write a situation equation to represent the relationship of the numbers in the problem.
Solve the Problem MP.5 Use appropriate tools. MP.8 Use repeated reasoning.	**Find the Answer** Children use the math drawing and/or the situation equation to find the unknown.
Check That the Answer Makes Sense MP.3 Critique the reasoning of others. MP.6 Attend to precision.	**Check the Answer in the Context of the Problem** Children write the answer to the problem including a label. They explain and compare solutions with classmates.

Problem Solving

Problem Types In this unit, children solve story problems involving situations of adding to, taking from, putting together, and taking apart with the unknown number in any of the three positions.

Add To with Result Unknown: Lessons 9, 10, 11

There are 4 birds.
Then 3 more birds join them.
How many are there in all?

Add To with Change Unknown: Lessons 2, 4, 5, 9, 10, 11

There are 6 camels at the zoo.
Then some more camels come.
Now there are 9 camels at the zoo.
How many camels come?

Add To with Start Unknown: Lessons 9, 10, 11

There are some swimmers in the pool.
2 more swimmers get in the pool.
Now there are 10 swimmers.
How many swimmers are in the pool at first?

Take From with Result Unknown: Lessons 6, 7, 8, 10, 11

We have 9 computers in our school.
Then 3 are moved to another school.
How many computers do we have now?

Take From with Change Unknown: Lessons 10, 11

There are 8 oranges.
Some oranges are eaten.
Now there are 5 oranges.
How many oranges are eaten?

Take From with Start Unknown: Lessons 9, 10, 11

There are some children playing.
Then 4 children leave.
Now there are 6 children.
How many children are there at the start?

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from **THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING**

An Algebraic Perspective Students thus begin developing an algebraic perspective many years before they will use formal algebraic symbols and methods. They read to understand the problem situation, represent the situation and its quantitative relationships with expressions and equations, and then manipulate that representation if necessary, using properties of operations and/or relationships between operations. Linking equations to concrete materials, drawings, and other representations of problem situations affords deep and flexible understandings of these building blocks of algebra.

Put Together/Take Apart with Total Unknown: Lessons 9, 11

> There are 5 blue toy dinosaurs
> and 4 green toy dinosaurs.
> How many dinosaurs are there?

Put Together/Take Apart with Addend Unknown: Lessons 2, 3, 4, 5, 8, 9, 11

> There are 8 children in the park.
> 3 are swinging. The rest are on the slide.
> How many children are on the slide?

Put Together/Take Apart with Both Addends Unknown: Lessons 2, 4, 5

> Anthony has 10 fish.
> He wants to put some fish
> in each of his 2 bowls.
> How many fish can he put in each bowl?

The three-part structure of Add To/Take From and Put Together/Take Apart problems are designed to support the fact that the unknown can be any part of the equation.

$6 + 2 = \square$ $6 + \square = 8$ $\square + 2 = 8$

Children need to understand the situation to know which problem type it is. We shouldn't use "keyword" methods where a given word tells what operation to use. Children must understand the whole problem structure.

Add To/Take From problems involve some action. Put Together/Take From problems involve two quantities that, together, compose a third quantity.

Focus on Mathematical Practices

Lesson 12

The Standards for Mathematical Practice are included in every lesson of this unit. However, there is an additional lesson that focuses on all eight Mathematical Practices. In this lesson, children use what they know about addition and subtraction to solve problems about sports.

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from THE PROGRESSIONS FOR THE COMMON CORE STATE STANDARDS ON OPERATIONS AND ALGEBRAIC THINKING

Problem Types Add To/Take From situations are action-oriented; they show changes from an initial state to a final state. These situations are readily modeled by equations because each aspect of the situation has a representation as number, operation (+ or −), or equal sign (=, here with the meaning of "becomes," rather than the more general "equals").

In Put Together/Take Apart situations, two quantities jointly compose a third quantity (the total), or a quantity can be decomposed into two quantities (the addends). This composition/decomposition may be physical or conceptual. These situations are acted out with objects initially, and later children begin to move to conceptual mental actions of shifting between seeing the addends and seeing the total (e.g., seeing children or seeing boys and girls, or seeing red and green apples or all the apples).